

Lesson Plans 1.1

Hands-on activities for students of

Elementary Grades through High School.

Mathematics

Art

Science

Architecture

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Zome System in the Classroom

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Zome System

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Zome System in the Classroom

Subjects addressed by Zome System

The relationships inherent in Zome System's components make it possible to teach a vast number of concepts. Teachers can apply the system to both purely mathematical pursuits, as well as a tool to explore structure and geometry as they relate technology, fine arts and the natural sciences. While a few of these concepts are covered by the initial lesson plans in this document, many more can be illustrated with Zome System. A partial list of ideas include:

- Art: proportions and scale, symmetry, perspective drawing, mosaics, and architectural design;
- Biology: symmetries in fruits and flowers, Fibonacci sequences, cell structures; honeycombs, DNA structures;
- Chemistry: basic molecules, Buckyballs and other complex molecules;
- Civics: city planning; resource distribution, role of art and design in society;
- Geology: crystallography
- Mathematics: geometry, numbers sense, progressions, algebra, trigonometry, Fibonacci sequences, budgeting;
- Physics/technology: perspective drawing, architectural structures, stresses and gravity forces, surface tension.

Organization of Plans

The lesson plans are divided into three groups based on complexity level; "basic concepts," "intermediate concepts," and "advanced concepts". The complexity level is noted at the top of each plan. The three complexity levels are related to, but not limited to, the grade level the plans are appropriate for. It is quite possible that we have miscalculated the appropriate complexity level for some of the plans. Teachers should evaluate the plans individually before introducing the concepts to their students. The intent of the complexity classification is that first through third grade students will be able to grasp the easier "basic concept" plans. Fourth and fifth graders will be able to work with all the "basic concept" plans, and in many cases move into the "intermediate concepts." Middle school, and high school teachers should review the "basic concept" plans before moving into the "intermediate" and "advanced" plans. The lessons "*Shape and Number*," "*What is Reflection Symmetry?*" and "*Multiple Reflection Symmetry*" are particularly important for all grade levels to have covered before advancing. We encourage all teachers to try concepts that they might not have attempted with their students in the past. Zome System is a very powerful system, and even very young students can use it to understand advanced ideas.

Related lessons are organized in "strings," where a plan may build on concepts introduced in a previous one, and suggest continuation in more advanced ones. Here it is also important that the teacher ensures that students understand the "basic" ideas and how they work in Zome System, before they start working on the "intermediate" and "advanced" plans.

Graphics

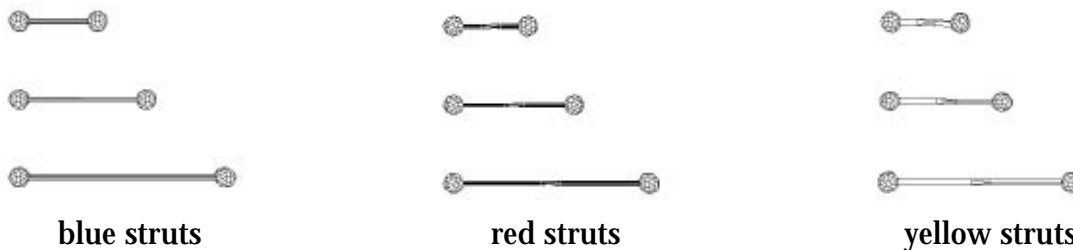
Most of the plans in Zome System Lesson Plans 1.0 contain graphics of Zome System models. The models are either required in the plans, or are examples of structures students may build during the class. To contain printing costs these graphics are in black and white. However, the graphics are

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in most cases easy to interpret, as the shape of the struts is indicated. The struts can also be identified by their shading; red struts are depicted as dark gray/black, blue struts as light gray, and yellow struts as white. None of the graphics in this book depict the supplementary green struts that are available from Zometool, Inc. Green struts are necessary to build some specific 3-D solids.



Standards and Assessment

To aid teachers in states or provinces that promote uniform achievement levels, the lesson plans make reference to educational standards. Whenever possible we have chosen standards that are developed or recognized by US national organizations in the subject area. In the case of mathematics concepts this means the standards adopted by the National Council of Teachers of Mathematics (NCTM). As most state and district standards in the US draw heavily on their national counterparts, American teachers will find that they can relatively easily convert one to the other. References to standards on the lesson plans is limited to the title of each standard. More extensive text, including the various benchmarks the plans address, is available as an appendix at the end of this document.

The “Assessment” section of each plan suggests how a teacher can evaluate students’ understanding of the concepts they are working on. The assessment model is roughly in line with the one featured in the NCTM Assessment Standards. The section states what the students must achieve or create in order to meet the standard, and what they must do to exceed the standard.

Preparing for Class

Play with Zome System before teaching with Zome System! The system has so many possible combinations and possibilities that teachers will benefit from exploring before introducing it to students. Playing with the pieces in advance will make the lesson plans easier to relate to. Students will invariably ask questions about the system, and discover new relationships that can serve as a basis for additional classes.

The need to play and explore also applies to students. The first time Zome System is introduced, students should be allowed to investigate the system without having a specific task to perform. The initial exploration will help prepare them for the classes, and will make them excited about the concepts they are to pursue. It is of course appropriate to discuss any features and relationships students observe during the “play period”.

Discovery Learning with Zome System

The lessons in Zome System Lesson Plans 1.0 are written in a format that promotes discovery

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learning. In most cases the plans are structured so the students are given tasks to complete, or problems to solve using the Zome System pieces. Discussion questions are suggested in italics to prompt student analysis of the concepts they are working on. The intention is that students will build understanding for themselves so they can explain and replicate the knowledge. This process stimulates student curiosity and encourages them to study the subject matter further. Teacher will also find that the lessons will allow them to discover new ideas and challenge their previous understandings.

The discovery learning promoted here does not replace the need for direct instruction. Direct instruction is needed for clarification and guidance throughout each class. The level of instruction required for each group of students should be evaluated by the teacher.

Cooperative vs. Individual Learning

Most of the lessons in this document encourage group projects. Work in groups promotes valuable team skills that often are difficult to apply in mathematics and science courses. By discussing the challenges they are given in the lessons, students practice communication in mathematical language, and draw deeper understanding from the observations of their team members. A useful discussion on the benefits of group work can be found in the “discourse” section of the NCTM Professional Teaching Standards.

Zome System can of course be equally useful for individual student projects. The lessons can easily be modified to for individual work as deemed appropriate by the teacher. The lessons may in some cases require larger numbers of Zome System pieces when students work individually.

Working with Diverse Student Groups

Zome System is unusual in that it can be applied with excellent results at a wide range of proficiency levels. No previous knowledge is required to explore various concepts with the system. Gifted students with strong mathematical, logical, and visual/spatial skills are very attracted to Zome System. These high achieving students use Zome System to make advanced connections between mathematics, technology and natural science. Students at the other end of the academic spectrum are also helped greatly by working with Zome System. By building structures, these students get to use more of their senses to understand a concept. They get to hear the teachers explanation, see how the number and shapes relate to each other, and get to touch and manipulate the concept. Virtually all students benefit from the active involvement Zome System promotes. One of our lesson plan writers commented on how exciting it was to see her students immersed in their tasks. Even her identified ADD (Attention Deficit Disorder) student was so engrossed that he never got up from the building area for the entire math period.

Activities with Zome System do not require specific levels of language comprehension. The concepts the students are exploring are universal and accessible to all, regardless of cultural background. The Weld County 6 School District in Colorado is currently using Zome System in all of their English as a Second Language (ESL) classes as a way to promote language skills. As the students grasp the universal geometry concepts, they also practice the meaning of the words the teacher is connecting with their structure.

Teachers will find that their students are very proud of the shapes and structures they create, and

will like to have them displayed. In fact, the most difficult task for the teacher is often to get the students to break down their structures at the end the class. If the Zome System kit is not going to be immediately used for another lesson, the teacher can let students hang their structures from the ceiling in the classroom as a visual reminder of the concepts they have learned. If this is not possible, it is advisable to have a camera on hand to document everybody's achievements.

Additional Resources

The lesson plans in this document should only be seen as a first step in exploring Zome System in the classroom. Teachers should also review the support materials that are included with the various Zome System kits. Manual 2.0, the full-color brochure that comes in all the kits, describes how the Zome System system works, including how the different elements relate to numbers and symmetries in space. In spite of its small size this manual contains a very large amount of information. Teachers can turn many of these ideas into full scale classes, as well as problem solving exercises for individual students. Manual 2.0 also contains instructions on how to build numerous geometric and architectural models. Zometool Manual, which is included in the *Zome System Explorer Kit* and *Zome System Creator Kit*, contains math activities for primary school through college. Math teachers are strongly encouraged to implement its ideas in their classrooms. Additional copies of the above documents can be obtained by calling Zome System toll-free at 888-966-3386.

Zome System will be producing many more lesson plans and activity books in the years to come. Teachers with ideas for classroom applications are encouraged to contact us, as we will commission plans from educators who are using our tool. Other textbooks and support materials will be produced by various educational publishers.

It is important to remember that many activities in textbooks that are currently in the market, can be easily adapted to work with Zome System. Please see the "Resources" section of this document for a few suggestions. Zome System can be used in conjunction with, and as a reinforcement of, other manipulatives and software programs (i.e. geoboards, tangrams, Geometer's Sketch Pad etc.) Teachers will find that many of the lessons that they have developed over the years can also be taught using Zome System.

Caring for your Zome System Kit

Zome System components are made of high quality ABS plastic, and are very durable. The struts are designed to flex slightly to make it possible to insert them into tight structures. Zome System structures will come apart when exposed to heavy loads (see for example "*Tallest Tower in the World*"), but the pieces will remain unaffected. Dirty parts can be washed in a garment bag in a clothes washing machine. Please note that ABS plastic is affected by ultraviolet light. It is therefore not advisable to expose Zome System parts to sunlight for extended or repeated periods of time, or they may become brittle or faded. However, short-term (<1 hour) or intermittent exposure to sunlight will not damage the pieces.

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Mathematics Basic Concept

Lesson Objective:

Students will learn to identify a **polygon** from its number of sides after looking at a model or a picture.

Prerequisite Skills:

Students need to have played with Zome System before.

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

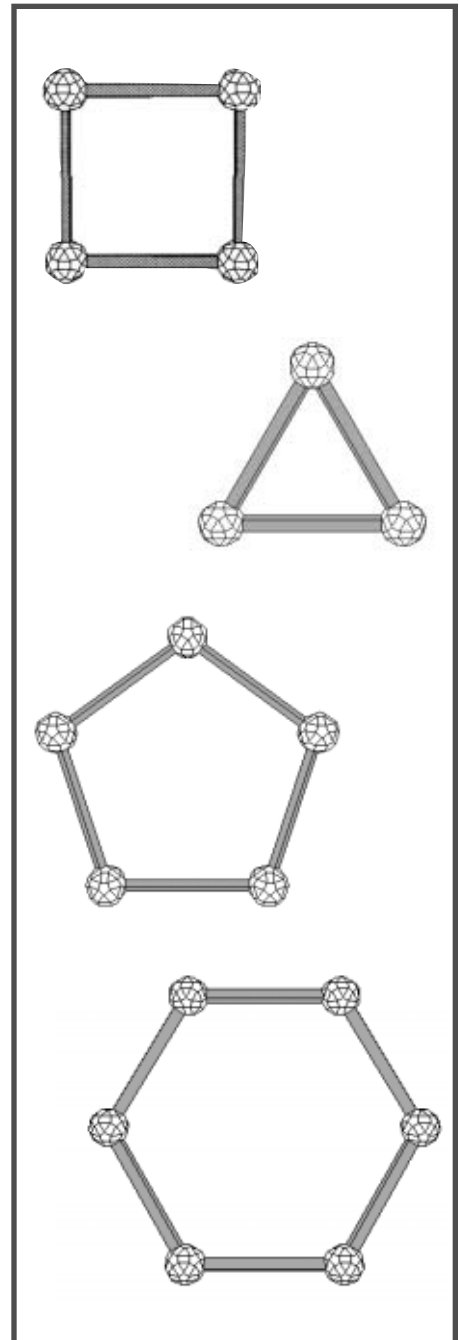
- One Zome System Creator Kit for 25-30 students
- Shapes by Ishtar Schwager Publications (choose another book about shapes if this one is not available)

Procedure:

Read your book about geometric shapes to the class. Discuss the different shapes and their properties introduced in the book. *How many sides does the shape have? Does the shape look like anything else we know?*

Distribute the Zome System pieces, and give 20 minutes to try to build some of the shapes in the book. Bring the class back together for a discussion. Ask each student to introduce one shape that they built. *What shapes they were able to build? Why couldn't they make a circle? How can we organize the shapes into groups? Which shapes look alike?* Guide the students towards a system where shapes are grouped according their number of sides. (For this activity, put all squares and rectangles in the set with four sides.) Record the findings on the board or on chart paper. Example: **triangles** have 3 sides; **squares** have 4 sides; **rectangles** have 4 sides; **pentagons** have 5 sides; **hexagons** have 6 sides.

In a variation for young students, the class can be divided into cooperative teams with specific job assignments.



Geometric Shapes

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For example; one group would be responsible for making triangles, one work on squares, etc. Older students (i.e., second graders) can draw pictures and write about the properties of the shapes they make in their math journals. Display the shapes that were built, around the classroom.

Assessment:

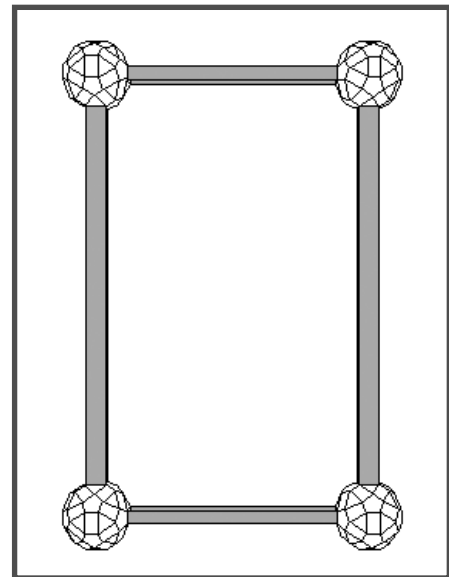
Question students individually and in groups. Ask them to build shapes with Zome System, and name them correctly. Students have met the standard if they can identify basic polygons from their number of sides.

Standards Addressed:

* Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

More work on polygons (“Geometry is all Around Us”). Exploration of number relationships and symmetry concepts in polygons (“Shape and Number,” “What is Reflection Symmetry?” and “Multiple Reflection Symmetry”). Expansion into 3-dimensional forms (“2-D and 3-D Shapes”). Use of geometric shapes in buildings and other man-made structures (“Tallest Tower in the World,” “Bridge Building Unit”). Art and design applications (“Trying Tessellation”).



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Geometry Is All Around Us

Mathematics Basic Concept

Lesson Objective:

Students will increase their familiarity with various polygons.

Prerequisite Skills:

Knowledge of basic 2-dimensional shapes ("Geometric Shapes"). Ability to define geometry.

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

- One or two Zome System Creator Kits for 25-30 students.
- The Shapes Game by Paul Rogers (1989, Henry Holt and Company, Inc.)

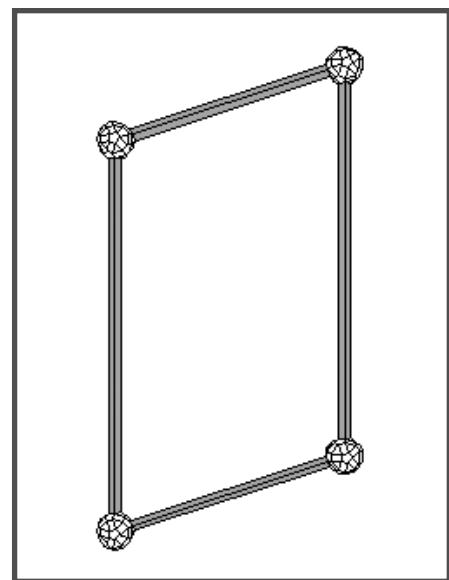
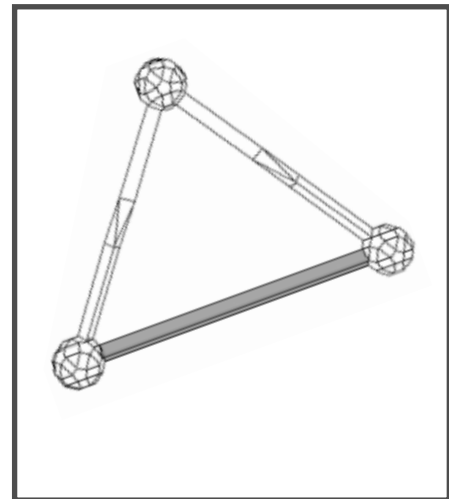
Procedure:

Begin the lesson by exploring your students understanding of what geometry is. *What is geometry? What subject or subjects does geometry belong to? Why is it important to study geometry?* Collect brainstormed ideas on the board or on chart paper. *Can we see any examples of geometric shapes in the classroom?* Add examples to the brainstormed list.

Read The Shapes Game to the class. Discuss the illustrations as the book is read.

The first challenge for the students is to predict which Zome System struts will be needed to create the shapes they saw in the book. The students can either work individually or with a partner. They should record their predictions in their math journals. Journal entries should include the number and color of struts they think will be needed for a certain shape.

Distribute the Zome System pieces evenly to the class. Students should attempt to build the shapes they have predicted on paper. As they build, they should record how



Geometry Is All Around Us

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the shape was actually built. Ask students to either make a presentation to the class, or to members of their cooperative team, about how their constructions differed from their prediction. Students should also write a letter with a picture and definition of the shape they created using Zome System. The letter should be written so a Kindergarten student could understand the shape described. The class can also write geometry riddles about various shapes. Share these aloud and have students guess the shape.

Hang completed shapes on walls and from the ceiling in the room as a visual reminder about shapes around us.

Assessment:

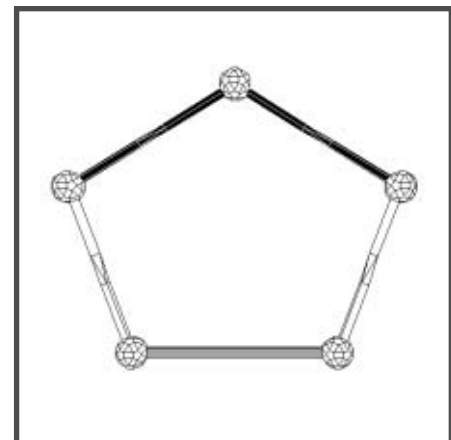
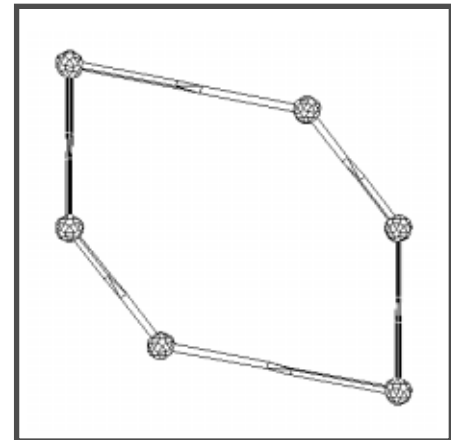
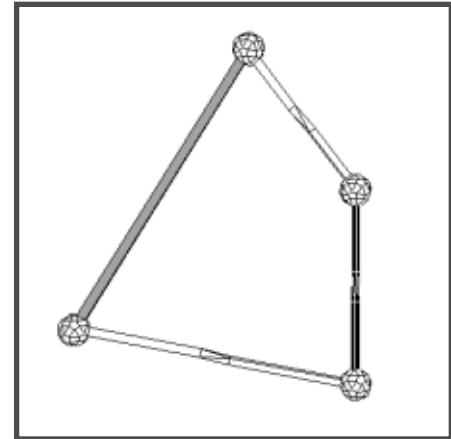
Take notes during discussions and presentations, and review journal entries and descriptive letters. To meet the standard, students must build and correctly describe at least four different polygons. To exceed the standard they must write definitions that are clear enough that another student could replicate the shapes without the use of a visual aid.

Standards Addressed:

* Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Exploration of number relationships and symmetry concepts in polygons (“Shape and Number,” “What is Reflection Symmetry?” and “Multiple Reflection Symmetry”). Expansion into 3-dimensional forms (“2-D and 3-D Shapes”). Use of geometric shapes in buildings and other man-made structures (“Tallest Tower in the World,” “Bridge Building Unit”). Art and design applications (“Trying Tessellation”).



Mathematics Basic Concept

Lesson Objective:

Students will learn to build, record, and define different types of polygons.

Prerequisite Skills:

Students need to have played with Zome System before.

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

- One Zome System Creator Kit for 25-30 students
- One recording chart per student

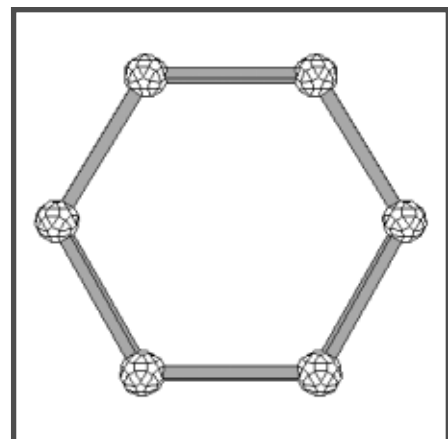
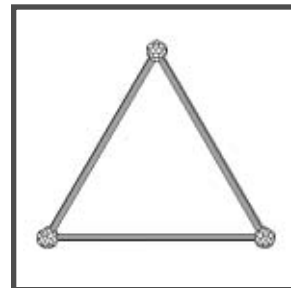
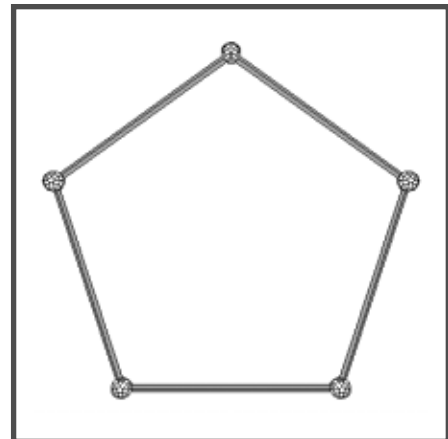
Procedure:

Prepare for the class by making charts where the students can record their findings about polygons. A suggested layout is presented below.

Polygon	Number of Nodes Used	Number of Struts Used
triangle	3	3
quadrilateral	4	4

The column headed “polygon” is for your students to draw and name the shapes they create. The two narrow ones will contain the number of Zome System elements used to build each shape.

Begin the lesson with a discussion about geometry. *Which shapes do you know the names of? What does that shape look like?* (Draw the shapes the students name on the board). *How do you know that this shape is called a square?* Continue questioning how they define each shape on the board until they establish that the number of sides determines the name.



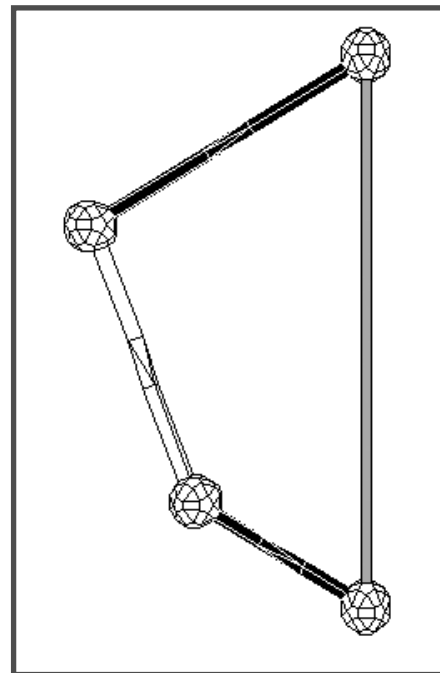
2-D Polygons

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Introduce the word “polygon”. *Does anyone know what a polygon is?* Explain that all the shapes previously named are polygons. *What do all the shapes on the board have in common?* Discuss this idea until it is established that polygons are closed shapes with two or more sides which can lie flat on the table (two dimensional space or plane).

Students will work in pairs, with their challenge being to create as many polygons as they can with the Zome System elements and record information about their work in the chart. Model how to record the number and color of struts used for each polygon. Red struts can be recorded as “R”, blue as “B”, and yellow as “Y”. Different triangles can thus be recorded as 3B, or 2R, 1B. Students must also draw pictures for each shape in the polygon column and name the shape if they can. If they are able to record the name for any of the polygons, do that as well. Encourage the students to build as many different polygons as possible - **rectangle, triangle, pentagon, square, hexagon, and decagon**. Students may not know the names for all shapes, particularly the hexagon and decagon. You may introduce the names for special triangles, such as **obtuse angle, equilateral, acute angle, and right angle**, during the discussion.



Find an example where two pairs of students made the same shape in different sizes. *Does this big blue shape have the same name as this little one?* Discuss how this can be so, to reinforce the concepts learned.

Assessment:

Observe students while they work. Question students individually and in groups to ensure that they can identify the different polygons by counting the sides of each shape. To meet the standard, students should understand the concept of polygon as related to the specific names for each shape. To exceed the standard they must build and positively identify squares, rectangles, triangles, pentagons, and hexagons.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

Expansion into more advanced shapes and 3-dimensional forms.

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Biology / Art Basic Concept

Lesson Objective:

To demonstrate similarity and difference in animal forms, including humans. Also to investigate geometric relationships within those forms.

Prerequisite Skills:

Previous experience building with Zome System. Knowledge of symmetry concepts is helpful (“What is Reflection Symmetry”).

Time Needed:

One or two class periods of 45-60 minutes.

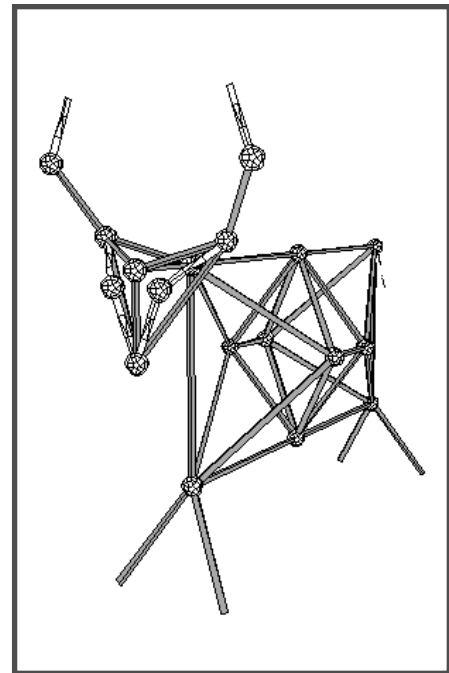
Materials Needed:

- Two Zome System Creator Kits for 25 - 30 students.

Procedure:

Introduce the lesson by stating that today we will focus on similarities among animals. *How are animals structured similarly? How do they resemble one another?*

Break the class into groups of 2-3 students, and challenge them to build an animal using Zome System elements. Encourage the groups to make different animals from each other. Give enough time so the models can be as detailed as possible. When the groups are finished, have them place their models on a table in front of the room, with a label stating which animal it is. Begin a discussion about the similarities and differences in each model. *What structural similarities exist in all the different animals modeled?* Encourage discussion until the concepts of bilateral symmetry, rotational symmetry, vertebrate function, and other anatomical characteristics are covered. If students don't use the correct vocabulary for these characteristics, introduce the new words. *Are there any recurring angular*



Animal Forms

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relationships in all of the animals? Roughly what are the angles? Are there differences in how the animals move? Do these differences appear to depend on any of the characteristics already mentioned? Are there any evolutionary advantages to being structured as they are? Why aren't all animals structured the same?

After the initial modeling, have the students develop models of imaginary creatures that demonstrate some of the general features all higher order animals possess, such as bilateral symmetry. Have a representative from each group explain the features of their imaginary animal to the class.

Assessment:

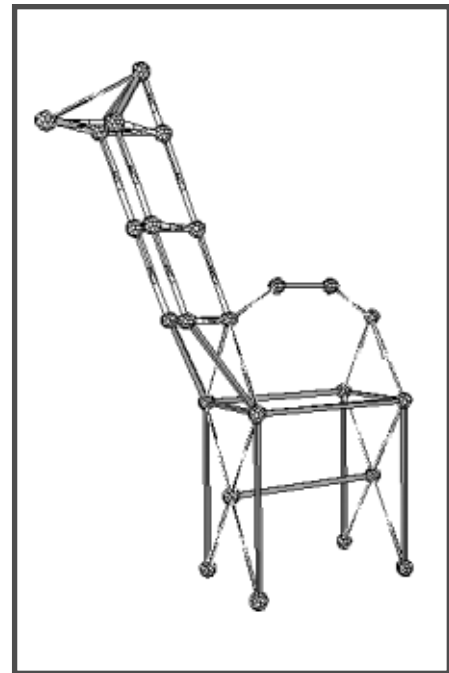
To meet the standard, students will be able to describe bilateral symmetry, rotational symmetry, vertebrate function, and other anatomical characteristics discussed. They will also understand how some features are adaptable for certain animals and not others.

Standards:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).
- * Science standards addressing animal anatomy and gravity forces.

Transfer Possibilities:

Exploration of symmetry and geometry in the natural world ("Multiple Reflection Symmetry").



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Attention!...Angles

Mathematics Basic Concept

Lesson Objective:

Students will increase their understanding of various types of angles. They will be able to build and manipulate angles using Zome System, and relate them to angles in their surroundings.

Prerequisite Skills:

Knowledge of what an angle is, ability to recall the three different types of angles

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

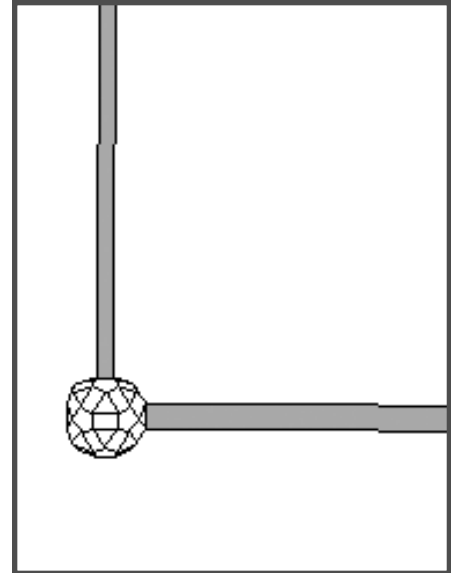
- One Zome System Creator Kit for 25-30 students.

Procedure:

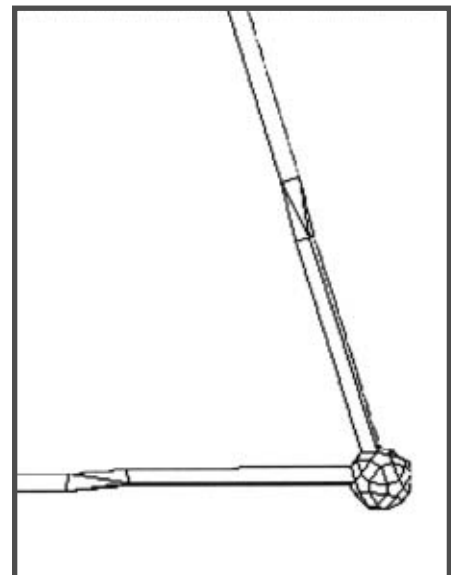
Divide the class into teams of 3-4 students, and distribute the Zome System pieces evenly. Their first project is to build any structure for 7-10 minutes. When they are finished they should study their model, with special attention to the struts. *What do you notice about the size of the struts? What do you notice about the directions they are pointing?* Compile a list of students' reactions.

From the brainstormed list, review the concepts of **parallel** and **perpendicular** lines. *How many parallel and perpendicular lines do you have in your structures?* The student teams should try to determine how many sets of parallel and perpendicular lines they have in their structure, and write them into their math journals. Have students notice example of these lines around the room.

Direct the students' attention to their structures and where the lines meet. Review the concept of **angles**. Have students recall what they know about angles and measurement.



right angle (90 degrees)



acute angle (<90 degrees)

Attention!...Angles

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What kinds of angles can you build with Zome System? What do parallel and perpendicular lines have to do with angles? Have students pick up a node and predict which struts will form an **acute**, **right** and **obtuse** angle. Record predictions in math logs. Then, have students build the three types of angles and draw them in their math journals. Ask them to compare observations with the members of their team. The drawings should be complemented with a definition for each angle type.

What angles do you see in the classroom? Can you identify a right angle in the room? Are there any obtuse angles? Any acute angles?

Assessment:

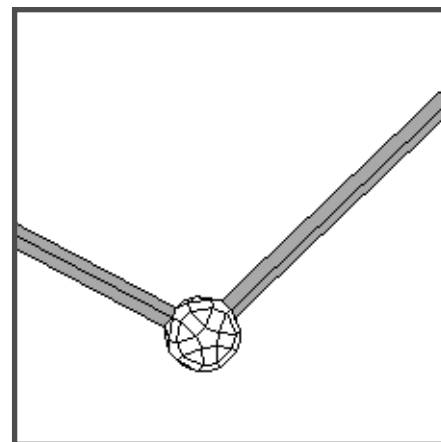
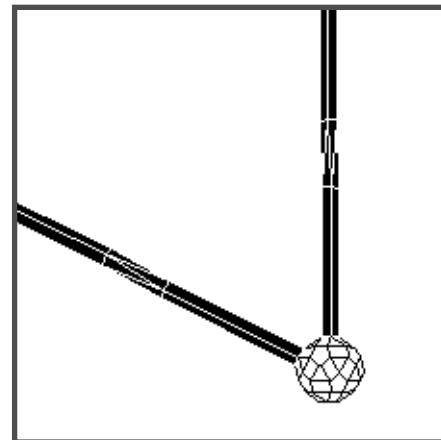
Observe students while they build angles. Review written work in math journals. To meet the standard, students must build, manipulate, and identify all three types of angles.

Standards Addressed:

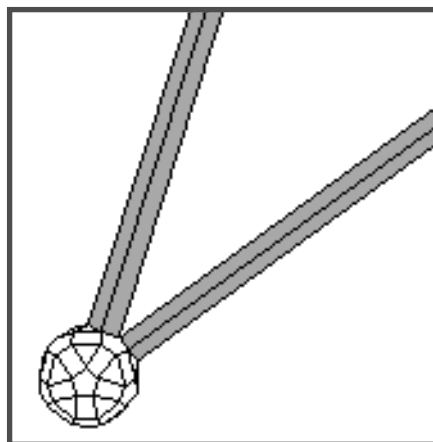
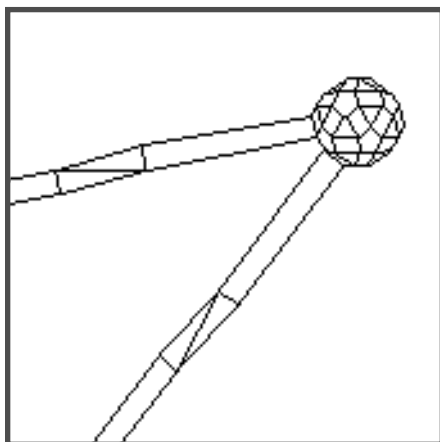
* Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

All future work involving angles in both mathematics and art.



obtuse angle (>90 degrees)



Zome System

Builds Genius!

Squares and Rectangles

Mathematics Basic Concept

Lesson Objective:

Students will learn the properties of squares and rectangles, and the difference between them.

Prerequisite Skills:

Students need to have played with Zome System before.

Time Needed:

One class period of 45 to 60 minutes.

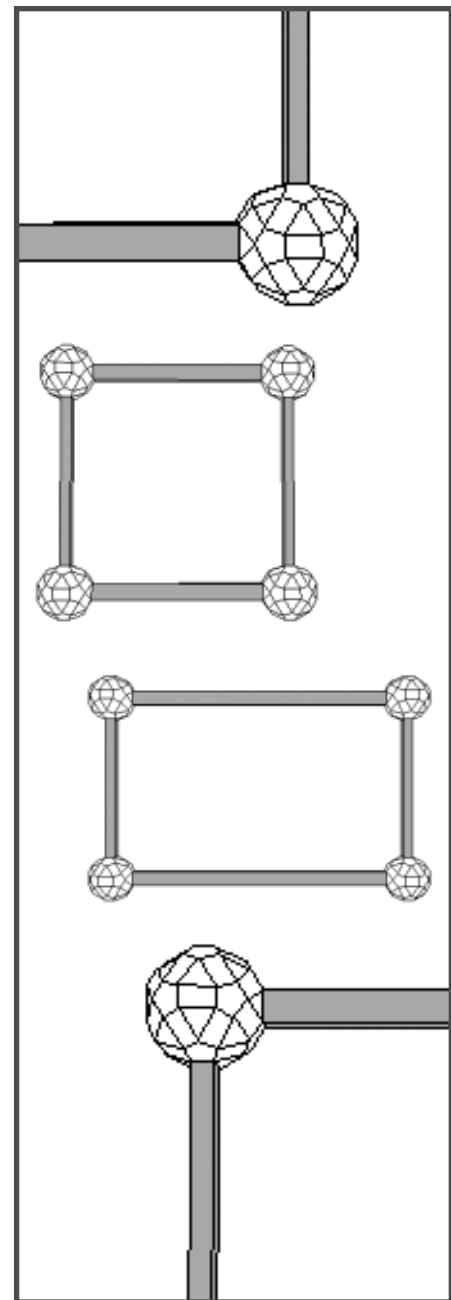
Materials Needed:

- One Zome System Creator Kit for 25-30 students

Procedure:

Prepare for the class by building a Zome System square and a rectangle. Show the students the square and the rectangle, and hold a short initial discussion. Record student ideas on the board or on chart paper. *How many sides do the shapes have? Are the two shapes exactly the same? What color struts were used? Does anything in the classroom have the same shape as one of the two shapes? What are the two shapes called?* Divide the class into pairs, and distribute the Zome System pieces evenly. The challenge for the student pairs is to build as many different versions of the two shapes as possible in a 15 minute period. They must also devise a method of sorting and classifying their shapes. Circulate and assist while they are working. Some students may build rhombuses, kites and other parallelograms, or they may build forms that are not plane.

Ask each pair of students to hold up a few of the shapes they have built to show the rest of the class. *How many different shapes did they find? Was it possible to build the shapes with all three colors of struts? Which different groups did they divide their shapes into? By size? By color? Was it possible to build the square in more than one size? In more than one color?*



Squares and Rectangles

Zome System

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Is a shape built with two long blue struts and two long red struts a rectangle or a square. Are the red and the blue struts the same length? Why was it not OK to build diamonds and other shapes with four sides? What is the same about squares and rectangles and what is different? Are the sides the same size or different? Are some sides the same and some different?

Record the similarities and differences pointed out by the students in a table on the board: Rectangles have four sides, two short sides and two long sides. Rectangles are long and skinny. Rectangles are not squares. Squares have four sides. Every side is the same length. Squares are not rectangles.

Older students (i.e., second graders) can instead write their findings directly in their math journals. All students can do drawings of a square and a rectangle.

End the class by again looking at forms in the classroom and have students determine if they are squares or rectangles.

Assessment:

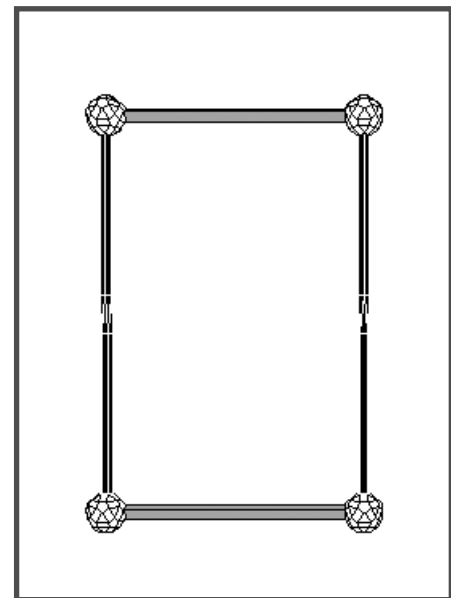
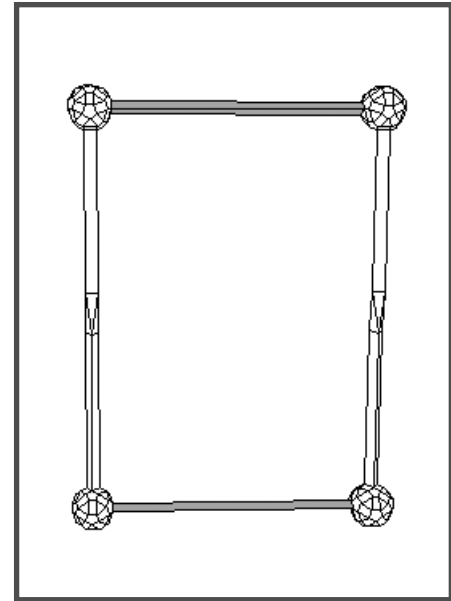
Ask students to build a square and a rectangle using Zome System and describe the properties of each shape. Their report can be delivered either orally or written. Key words/terms to look for: number of sides, same or equal, long or short. Students have met the standard when they can differentiate between the two forms. To exceed the standard they must verbalize definitions of squares and rectangles.

Standards Addressed:

* Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Expansion into more advanced shapes and 3-dimensional forms.



Zome System

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Mathematics Basic Concept

Lesson Objective:

Students will positively identify the three types of angles possible in a triangle. They will also learn to use a protractor

Prerequisite Skills:

Knowledge of the three types of angles, ability to identify triangles.

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

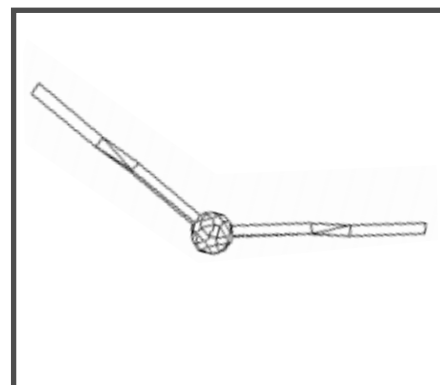
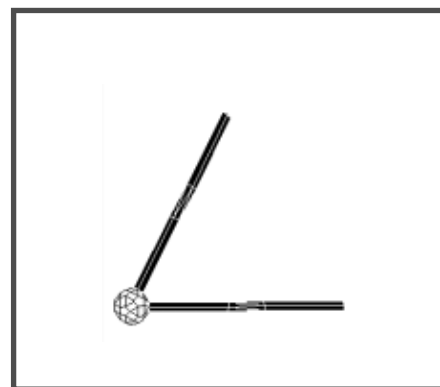
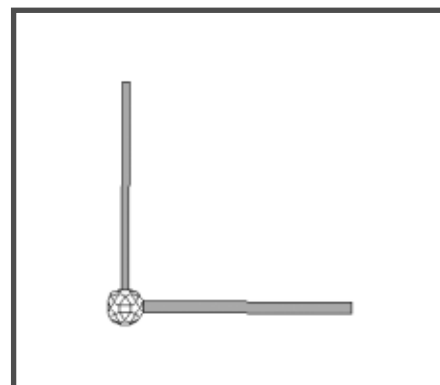
- One Zome System Creator Kit for 25-30 students
- Notes on a Triangle (video produced by Dale Seymour Publications and Cuisenaire Company of America, Inc.)
- One protractor per 3-4 students

Procedure:

Begin with a review of the different types of angles: **right**, **acute**, and **obtuse**.

View the video once and just let the students watch. When the video is over ask about their observations. *What did you notice? What is the name of the shape in the video? How do we know that a shape is a triangle?* View the video a second time and ask students to focus on the shapes that appear.

Distribute the Zome System elements once the video is completed, and ask the students to create as many different triangles as they can. All the triangles should be recorded in their math journals, including a sketch with the types of struts labeled. *How many different ones is it possible to build? Can you build them all in one color? Can you build any using all three colors?* Allow approximately 20



Try the Triangles

Zome System

Builds Genius!

minutes for experimentation.

When the students have completed their building time they are to share their results with teammates. The teams should sort their triangles into categories of their own choosing. *Do some of the triangles have anything in common? How can they be divided into groups?* Each team should make a short presentation of the categories they have chosen. As a class, decide what categories are constant throughout the class (i.e. right triangles, equilateral triangles, etc.). All students should enter the definitions into their math logs.

Discuss how someone would measure angles. *How do we know that two angles are the same or different? Can we guess, or is there a more precise system we can use?* Review the concept of a protractor. For some students the use of a protractor might be new. Discuss how the measurement of a triangle is equal to 180° . Each team should measure the combined angles on a few of their triangles and see if that rule applies to all triangles.

If time allows you can also introduce the concept of **congruency**. Hold a short discussion about the various triangles the students have built, and define the word **congruent** together.

Assessment:

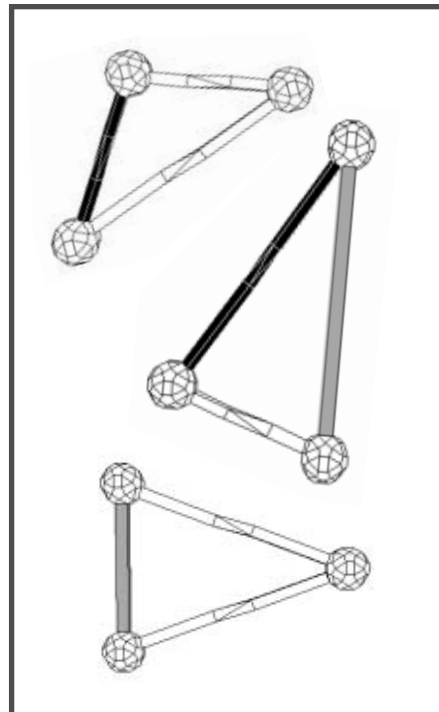
Observe students while they work. Question them to ensure that they can identify the types of triangles in the structure and measure the various angles. Review drawings and definitions in students' math journals. To meet the standard students must be able to show which angles in a triangle are acute, right, and obtuse. To exceed the standard they must correctly verbalize a rule showing how the combined angles of a triangle equals 180° .

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

More work on triangles ("Similar Triangles," and "Triangle Tiles"). Exploration of platonic solids and other 3-dimensional structures containing triangles ("Finding Plato's Solids I," "Finding Plato's Solids II," "3-D Triangles," and "3-D Triangle Tiles"). Work with triangles is also a natural lead-in to three-fold symmetries in geometry, art, and nature ("What is Reflection Symmetry?," and "Multiple Reflection Symmetries").



Zome System

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Mathematics Basic Concept

Lesson Objective:

Students will explore similar triangles using hands-on experience of the side-side-side law. They will learn the concept of congruency.

Prerequisite Skills:

Familiarity with Zome System and the concept of triangles (“Geometric Shapes”).

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

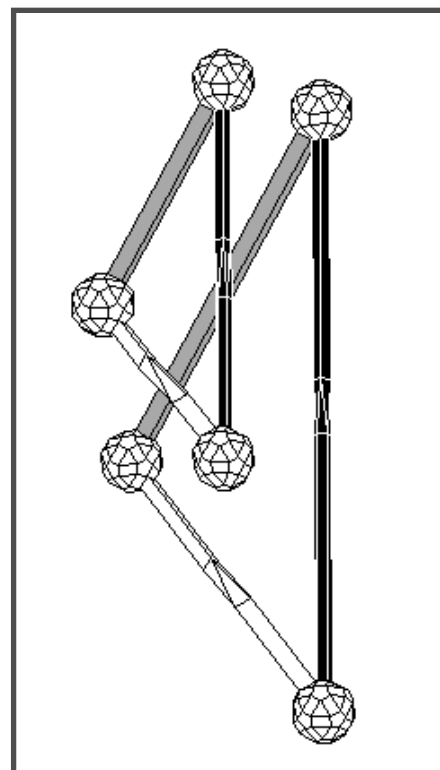
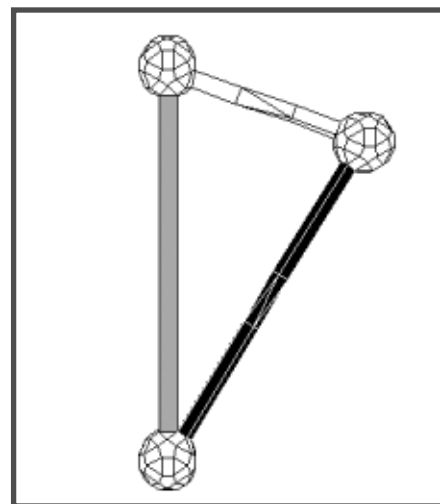
- One Zome System Creator Kit for class of 25-30 students
- A yardstick

Procedure:

Decide in advance on a unique triangle which can be built with Zome System, and write down the components on the board. For example:

- One long blue strut
- One long red strut
- One medium yellow strut
- Three nodes

Distribute the Zome System pieces evenly to the students. Explain that they are to build as many different geometric shapes they can using only the pieces in the “recipe.” The shapes must be closed so that every strut end is connected to another strut. *How many different shapes can we make with these 3 struts and 3 nodes?* Some students will be able to tell that only one specific triangle can be made. However, most will have to experiment to determine this. As your students build, walk around the class and collect



Similar Triangles

Zome System

Builds Genius!

all the triangles on a yardstick. Make sure to always hang them from the same vertex (the most acute angle works best.) The triangles hanging on the yardstick provides a very strong visual cue that the shapes are the same.

How many different shapes have the class made? Why did everyone build the same triangle?

Now repeat the exercise with a different “shape recipe” using a medium blue strut, a medium red strut, a short yellow strut, and three nodes. Long before you’ve picked up the last triangle, it will be obvious to the class that the triangles are “all the same, but two different sizes.”

Place the two sizes on the overhead projector and lead the class in a discussion of what happened: *Why are they all the same, but two different sizes? Why couldn’t anyone make a different triangle? What is the relationship between the struts in the little triangle and the struts in the big one? What if we had used 2 short yellow and one short blue strut? How can we be sure that two shapes are exactly the same?* Explain that two shapes that exactly overlap with each other are **congruent**.

Students should write down their observations in their math journals. They should also try to write a rule about triangles with similar sides. Any remaining time can be used to test the

rule using different Zome System components and making further discoveries. Older, or more advanced students, can try the exercise in 3 dimensions. Instead of triangles, they can build similar **tetrahedra** using 4 nodes and 6 struts each. *Does the rule hold in 3 dimensions as well?*

Assessment:

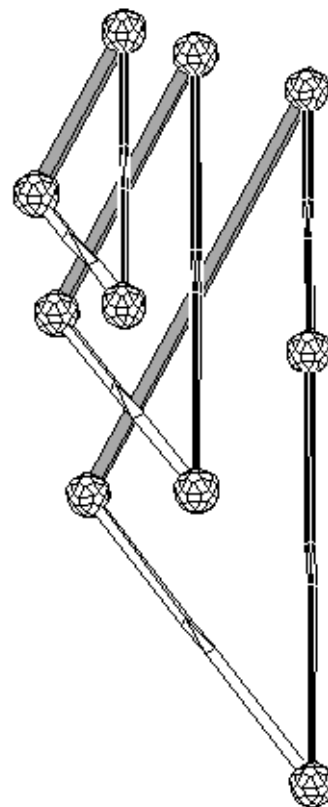
Take note of student ideas during discussions, and review individual work in math journals. To meet the standard students must determine that it is only possible to build congruent triangles when the edge lengths are set.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

Additional work on angles (“Attention!...Angles”) and deeper explorations of triangles and their mathematical and architectural uses (“Try the Triangles,” “Triangle Tiles - I,” “Triangle Tiles - II,” “3-D Triangles,” “3-D Triangle Tiles,” “Tallest Tower in the World,” and “Bridge Building Unit”).



Mathematics Basic Concept

Lesson Objective:

Students will be able to identify properties of triangles and learn the term **equilateral**.

Prerequisite Skills:

Knowledge of basic geometric shapes ("Geometric Shapes," "Shape and Number," and "Similar Triangles").

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

- One Zome System Creator Kit for 25-30 students.

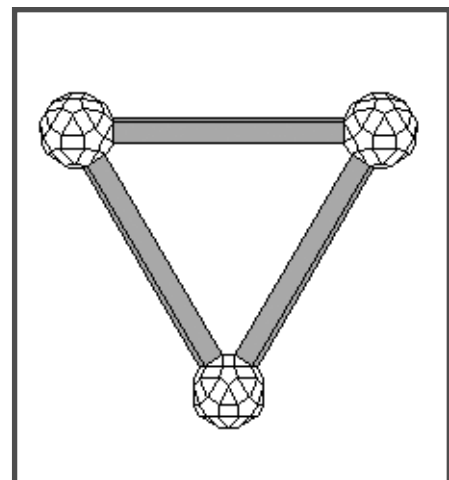
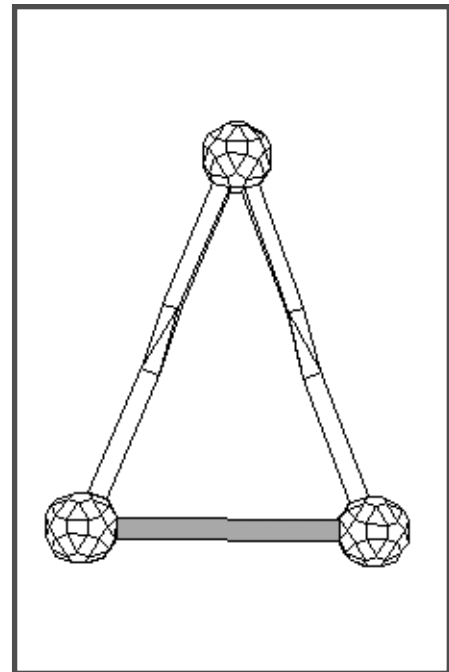
Procedure:

Prepare by building one triangle using 3 medium blue struts of the same length, one with 2 medium red struts and 1 medium blue strut, and one with 1 medium blue strut and two medium yellow struts. Write the "recipe" for these triangles on the board.

Divide the class into teams of 2-3 students, and distribute the Zome System pieces. Show the students the three different types of triangles. *Are they all triangles? Why are they triangles?* The challenge for the teams is to build the three triangles following the recipe on the board.

When all the teams have completed their work, their attention should be directed to the triangle made with 3 blue struts. *Are all the sides in this triangle the same length?* Ask the students to pick up three blue struts so that they can confirm that this is the case. Introduce the term **equilateral**, and explain that equilateral shapes have sides of exactly the same size.

Now show the class the red and blue triangle. *Is this is an equilateral triangle?* Most students will think it is because it looks very similar to the first triangle they saw. Point



The Equilateral Triangle

Zome System

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out the struts. *Are the struts the same color? Are all the struts the same size? How can we know for sure?* Ask the students to pick up a blue strut and a red strut and compare the sizes. The students will be able to see that the blue strut is longer, therefore coming to the conclusion that this is not an equilateral triangle.

Repeat the process using the yellow and blue triangle. Once again, discuss the struts and ask the class to decide if this is an equilateral triangle. Compare the lengths of all three color struts.

Allow the students to build more equilateral triangles. *Can they be built in other sizes? Is it possible to build equilateral triangles using only red, or only yellow struts (not without bending the struts)?* Students should describe the properties of the equilateral triangle, either orally or written. Older students (i.e., second graders) can be asked to draw an equilateral triangle using a ruler and a piece of paper. Discuss the concept of an equilateral shape. *Can shapes other than triangles be equilateral? Are some shapes always equilateral (the square)?*

Assessment:

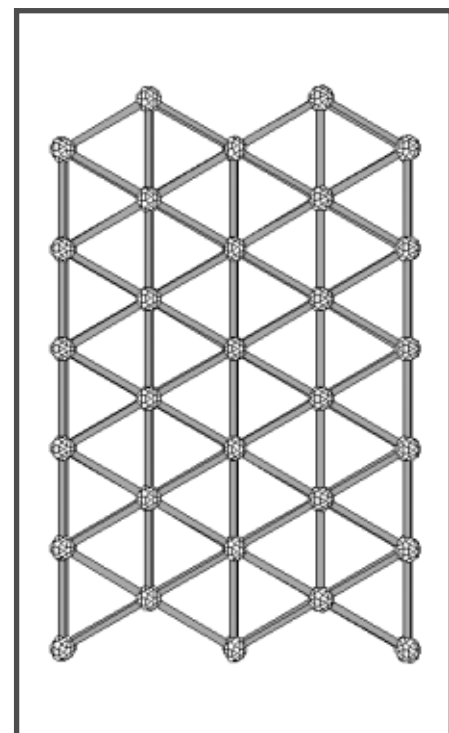
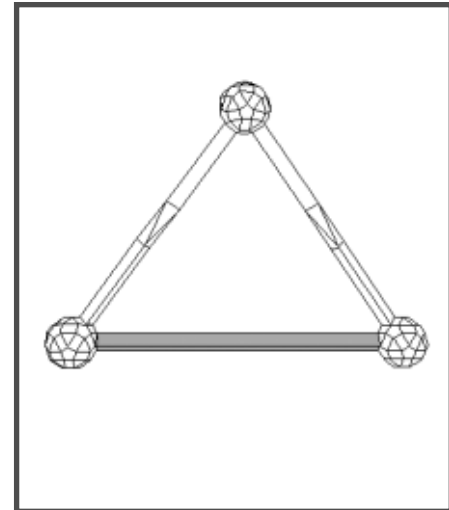
Observe students while they work, and review their notes and drawings. To meet the standard, students must build the three triangles, and clearly understand how the equilateral triangle is different than the two others. To exceed the standard they must determine that equilateral triangles can only be made using three blue struts.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

Expansion into more advanced 2-, and 3-dimensional forms, and their nomenclature ("2-D and 3-D Shapes," "What are Quadrilaterals?" and "Naming 2-D and 3-D Shapes").



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Mathematics Basic Concept

Lesson Objective:

Student will expand their knowledge of two-dimensional shapes, and learn how they can be changed into three dimensions. They will learn to build and draw models of **two-dimensional** and **three-dimensional** geometric figures.

Prerequisite Skills:

Familiarity with simple polygon shapes (“Geometric Shapes”).

Time Needed:

One or two class periods of 45-60 minutes.

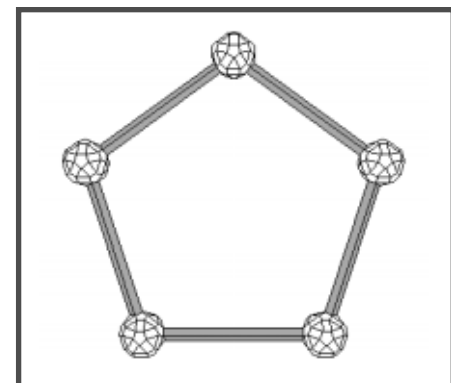
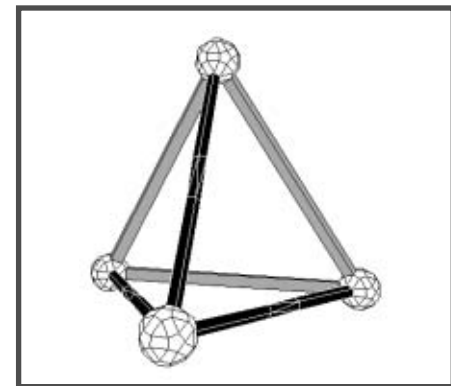
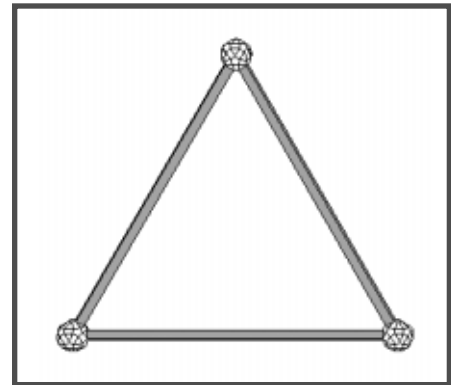
Materials Needed:

Two Zome System Creator Kits for 25 - 30 students

Procedure:

Start the class with a discussion about geometry. *Which geometrical shapes do you know the name of? How do you know that, for instance, a triangle is a triangle?*

Distribute a variety of Zome System elements to the students. Their challenge is to build two-dimensional shapes, for example triangles, rectangles, squares, pentagons, and rhombi. *How do we know that a shape is “two-dimensional”?* Discuss this concept until the class has determined that a two-dimensional shape must lie flat on the table so that all the nodes touch the table surface. Allow 10-15 minutes building time. Assess students’ ability to build a two-dimensional shape and identify the properties of that shape by having students present their shape to the class. *How can some of the triangles vary so much in appearance and still be triangles? How about the rectangles, squares, pentagons, rhombi, and other shapes?* Continue the



2-D and 3-D Shapes

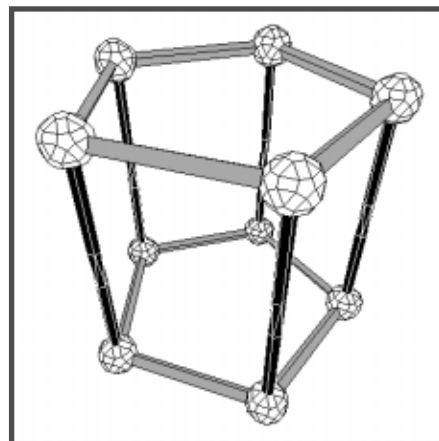
Zome System

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questioning until your class has agreed on definitions for the shapes they are working with. They should copy these definitions into their math journals.

Introduce any needed new vocabulary and concepts that will aid students in discussing their own constructions. New words may include; **Parallel Line**, **Perpendicular Line**, **Face**, **Edge**, **Vertex** and **Surface Area**.

Using the information learned from two-dimensional shapes, have students begin building three-dimensional shapes such as cubes, pyramids, and prisms using Zome System. The structures should be made up of combinations of the earlier identified 2-dimensional shapes. Have students compare their constructions with others from their group. Ask them to identify the faces, edges, and vertices of their structures.



Have the students draw their two-dimensional and three-dimensional shapes. Make a chart on the board or the overhead comparing the similarities and differences the students find in the two-dimensional shapes. Make another chart about the three-dimensional figures. *How and why do the shapes differ from each other? Why could certain shapes only be constructed with certain colored struts or combinations of struts?*

Finally, have students expand their knowledge of two-dimensional and three-dimensional shapes by constructing more complex three-dimensional figures. Identify the differences between the **pyramids**, **prisms**, and **polyhedra**. Have them identify what two-dimensional shape initially helped them to construct the shape. If possible, have students identify where they may have seen a shape like theirs. American students may, for example, identify the geometrical sphere at Epcot Center in Florida.

Assessment:

Observe students while they work. Question them while they present their structures to the class. Study student drawings.

To meet the standard students must build and name 4-5 basic polygons. To exceed the standard they must verbalize definitions for their polygons.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

More work with polygons (“What is Perimeter?,” “What is Area?,” “Attention!...Angles,” and “Try the Triangles”). Further exploration of geometric shapes in architecture (“Tallest Tower in the World,” and Bridge Building Unit”).

Zome System

Builds Genius!

Shape and Number

Mathematics Basic Concept

Lesson Objective:

Students will learn the relationship between the rectangle, triangle and pentagon and the numbers 2, 3 and 5, leading to a basic understanding of the relationship between shape and number. This concept is used extensively in Zome System lesson plans.

Prerequisite Skills:

Knowledge of basic polygons (“Geometric Shapes,” and “Geometry is All Around Us”).

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

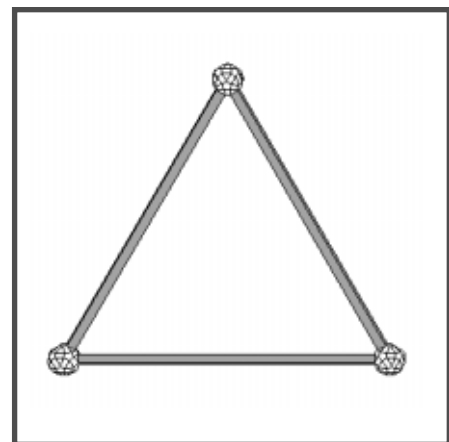
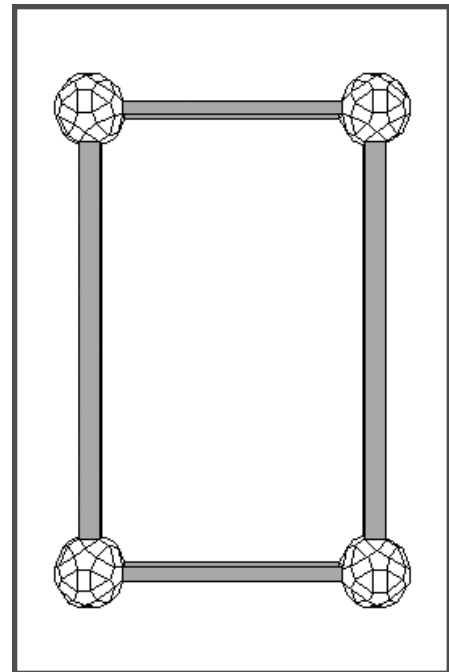
- One Zome System Creator Kit for 25-30 students
- Overhead projector

Procedure:

Divide your class into teams of 4 students each, and distribute the Zome System pieces evenly. Allow the students to explore freely with Zome System for 10-15 minutes. They should discover all they can about the Zome System system, discuss their findings with other team members, and individually write them down in their math journals. Circulate among the teams and help them explore and articulate their discoveries.

After the exploration, lead a class discussion. *What did you discover about Zome System? What are the shapes of the holes in the nodes? How can we be sure that a shape is a rectangle as opposed to a square? Which strut fits in which hole?* After a few minutes of discussion the class will agree on the names of the three shapes.

The next challenge for the class is to build the rectangle, triangle and pentagon using Zome System. Each stu-



Shape and Number

Zome System

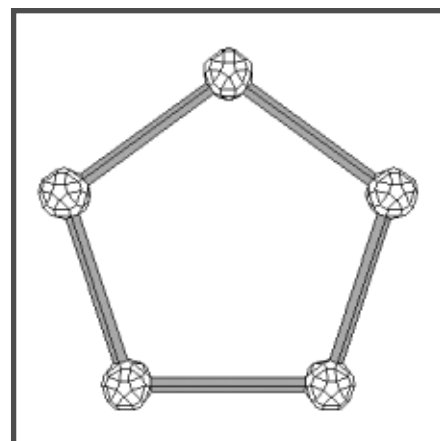
Builds Genius!

dent needs to build at least one of the 3 shapes: a **Golden Rectangle**, an **equilateral triangle** and an **equilateral pentagon**. Often these 3 shapes can be found embedded in structures built during the initial exploration. If necessary, help students make this connection, since it is generally more difficult to build the shapes in isolation. Students may attempt to build a triangle with the yellow struts or a pentagon with the red struts. This can result in interesting spirals, and students will discover Zome System's implicit rule: when it works, it works perfectly, but structures with struts bent, twisted or under tension are "illegal."

When a student or team completes a shape which they feel is the same as one of the holes in the node, place it on the overhead projector and ask for comments from the class. There will be general agreement if the shape matches; non-matches lead to discussion of squares vs. rectangles, **isosceles** vs. equilateral triangles, pentagons vs. hexagons, etc. Adjust vocabulary according to the age and experience level of the class; nomenclature is not as important as the concepts.

When all students have one of the 3 basic shapes, ask each to lay the shape flat on the table. Have them find a strut which fits into one of the holes in a node so that it stands **perpendicular**, or straight up from the table.

Discuss the relationship between the cross-sectional shape of the standing strut and the shape on the table (they are the same!) This feature of Zome System will help students begin discovering the relationship between shape and number. *What number might the rectangle represent? What number is the triangle? What number is the pentagon? How can we find out?* Have students count the number of points (nodes) and the number of lines (struts) in their shapes to formulate an answer. The students will quickly determine that the pentagon and the triangle are the number 5 and 3 respectively (5 identical lines, 5 nodes, 5 angles, 5 lines of symmetry). Many will feel that the rectangle is the number 4, since it has 4 points and 4 lines. In fact, it is more closely related to the number 2, i.e. it has 2 long struts, and 2 short struts, and only 2 lines



Zome System

Builds Genius!

of symmetry. At this point, it is sufficient to offer the square as a “better” number 4 because all its lines are the same lengths, and ask if any other number would work for the rectangle.

Conclude the lesson with a discussion of the use of symbols. *Which symbol or sign better represents the concepts of 2, 3 and 5; i.e. the written number “3” or the triangle?*

Assessment:

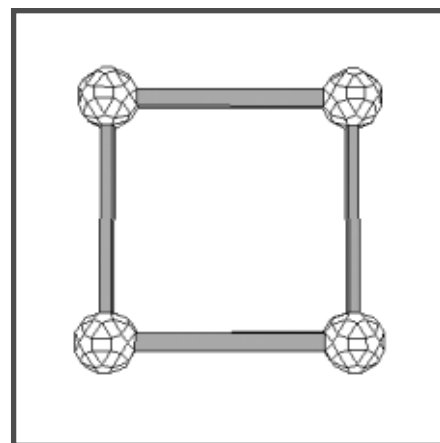
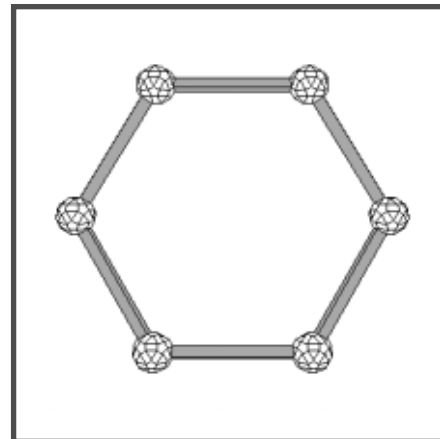
Observe students while they work, and review notes in math journals. To meet the standard students must build the three basic polygons and understand how they relate to the numbers 2, 3, and 5.

Standards Addressed:

- * Mathematics standards addressing **number sense and numeration** (NCTM Standard 6).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Exploration of symmetry concepts and number patterns in nature and the built environment (“What is Reflection Symmetry?” “Multiple Reflection Symmetry,” “Rotational Symmetry,” and “Fun Fibonacci”). Use of geometry to understand other mathematical concepts (“Odd and Even Numbers,” and “Prime Factors”).



Zome System

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What Is Reflection Symmetry?

Mathematics Basic Concept

Lesson Objective:

Students explore the concept of **bilateral reflection symmetry**, and discover its relevance in geometry and nature.

Prerequisite Skills:

Students need to have worked with Zome System before.

Time Needed:

One class period of 45 to 60 minutes.

Materials Needed:

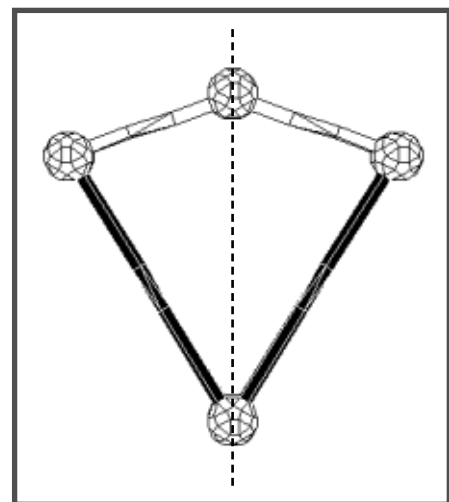
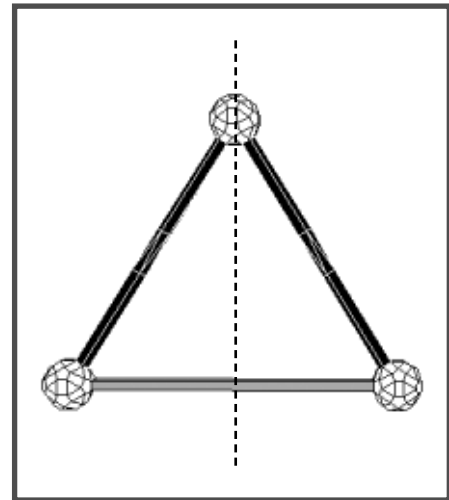
- One Zome System Creator Kits for 25-30 students
- A hand-held mirror

Procedure:

Ask the students what they know about **symmetry**, and **symmetrical**. Discuss the suggested definitions, and see if the class can agree on one or several options. *How can we tell if an object in the classroom has symmetry?*

Use a volunteer as a visual example for a discussion on symmetries in the human body. *What is a line of symmetry?* If we divide a symmetrical object, the line down which we divide or cut that object is called the line of symmetry. *If we could divide the student down the middle, would the two halves be symmetrical? Which way could we divide him/her so that the two halves would be the same?* Lead a discussion of how our bodies are not perfectly symmetrical due to uneven facial features, asymmetrical haircuts, and placement of internal organs.

Ask the students how to place a mirror on a student's face to demonstrate the reflection symmetry of the face and body. *What does the reflection show?* Half of the face plus its reflection gives a whole face. *Is this the same as the whole face?* Ask the students to write their own definitions for reflection symmetry. *Is it possible to divide the human*



What Is Reflection Symmetry?

Zome System

Builds Genius!

body into symmetrical halves, in more places than down the center? They will find that we only have one **axis of symmetry**. This is known as **bilateral symmetry**.

Distribute Zome System pieces to the class and ask them to build structures with reflection symmetry. They can build either 2-D polygons, or more complex 3-D structures. Have them work in teams of four to create one example to present to the class. They should record anything they discover along the way in their math journals. Depending on their age and spatial ability, some students will realize that shapes often have more than one axis of symmetry (they are not bilateral). List on chart paper or blackboard the properties of bilateral symmetry. *Do animals have bilateral symmetry? Would an animal be able to move efficiently if they were not symmetrical?* Make two columns on the board, labeled “bilateral symmetry” and “no bilateral symmetry.” *Can anyone name animals with bilateral symmetry? With no bilateral symmetry? With multiple symmetry?* Lead a discussion about how all of these animals move.

Do any man-made structures, such as buildings, have bilateral symmetry? Why are many buildings symmetrical? What kind of symmetry is it? What is the purpose of symmetry? Use the students’ structures as examples whenever possible.

Assessment:

Observe and listen to students as they build their structures. Older students should write about their findings in their math journals.

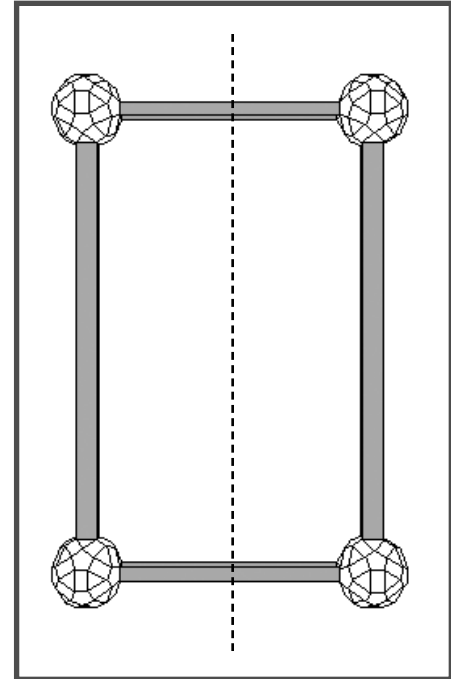
To meet the standard students must build a symmetrical model and show an axis of symmetry. To exceed the standard they must verbalize a few qualities/advantages of symmetry in an animal or natural object.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

More advanced work on different symmetry concepts (Multiple Reflection Symmetry,” “Rotation Symmetry,” “Translational Symmetries in Tilings,” and “Spiral Symmetries”). Human and animal anatomy (“Animal Forms”), structure and balance in botany, aesthetics in man-made structures, as well as many concepts in fine art.



Zome System

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Multiple Reflection Symmetries

Mathematics Basic Concept

Lesson Objective:

Students will understand how an object can have more than one line of reflection symmetry. They will find examples of these symmetries in the man-made and natural worlds.

Prerequisite Skills:

Students should be able to name basic polygons (“Geometric Shapes”), and be able to find lines of symmetry in a Zome System model (“What is Reflection Symmetry?”).

Time Needed:

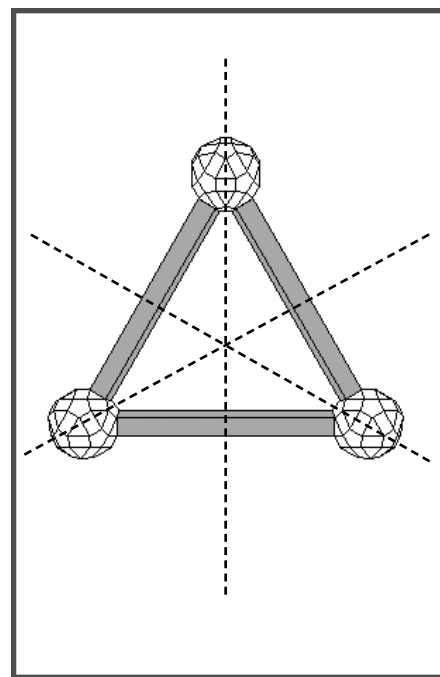
One or two class periods of 45 to 60 minutes.

Materials Needed:

- One or two Zome System Creator kits for 25-30 students
- A few natural objects displaying 3-fold symmetry (banana, green pepper, clover, cucumber, flowers with 3, 6, or 9 petals, honeycomb, snow flake, etc.)
- A few natural objects displaying 5-fold symmetry (apple, pear, zucchini, starfish, sand dollar, flowers with 5 or 10 petals, maple leaf, etc.)
- A hand-held mirror

Procedure:

If you work with younger elementary students you may want to cut the fruit and vegetables beforehand. Older students can do their own cutting. The fruit and vegetables should be cut on their equators to reveal their internal symmetry/number of sections. Alternatively you can use one of the many posters or other pictorial depiction of “numbers in nature”, that are available from educational publishers.



Multiple Reflection Symmetries

Zome System

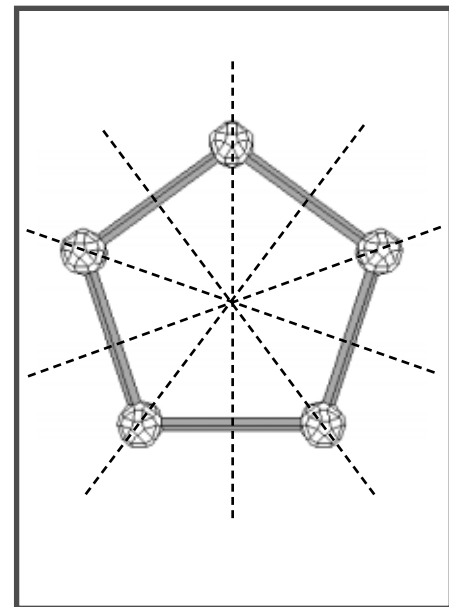
Builds Genius!

Divide your class into teams of 3-4 students, and distribute the Zome System elements. Review the concept of reflection symmetry they learned in the class “What is Reflection Symmetry”. Tell the teams that they are going to build and find symmetries in the following polygons: a square, a rectangle, an equilateral pentagon, an equilateral triangle, and an equilateral hexagon. The names of the polygons should be written on the board. While they have found one axis of reflection symmetry in the previous class, this time they are going to see if the assigned polygons have more than one axis. *Is it possible to divide a shape in more than one direction and get symmetrical halves?* Discuss this concept as necessary.

Walk around and help the students while they work. Allow them to use the mirror to check that they really have chosen axes of reflection symmetry. The visible half of the polygon along with its reflection in the mirror will show an image of the entire polygon..

Bring the results from each team together on the board. *How many lines of symmetry have they found in each polygon? Is there a limit on how many lines of symmetry an object can have?* Discuss the results and let students present their findings. Students will realize that the number of symmetry lines is the same as the number of edges in the equilateral polygons. This does not hold true for the rectangle. *Do other objects have reflection symmetry, for instance the letter “H”? What is a good name for each type of symmetry?* Help the group settle on names, the triangle can for instance be said to have 3-fold, 3-directional, or triangular symmetry.

Next turn to the natural objects you have brought in. You can either let the students search for symmetries, or you can display them to the entire class. *How many sections do they have? Why do the number 3 and 5 come back so frequently? How do the 6-fold symmetries of honeycombs, snowflakes, and many flowers relate to 3-fold symmetries ($3 \times 2 = 6$)? How do the 10-fold symmetries in a pumpkin or an acorn squash relate to 5-fold symmetries ($5 \times 2 = 10$)? Which symmetries can they find in the classroom?* Check the students comprehension by letting them show the 3-, and 5-fold



Zome System

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Multiple Reflection Symmetries

symmetries of the natural objects with the help of the two Zome System “stars” in the graphic on the right.

Assessment:

Observe and listen to students as they build their structures. Older students should write about their findings in their math journals.

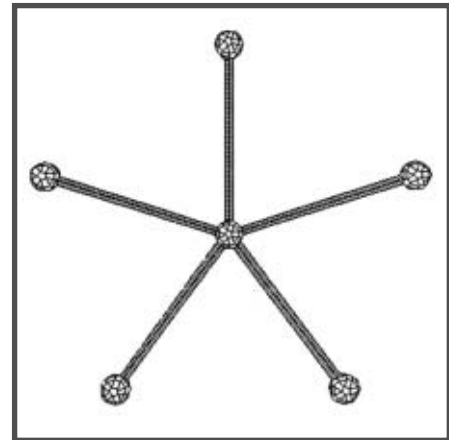
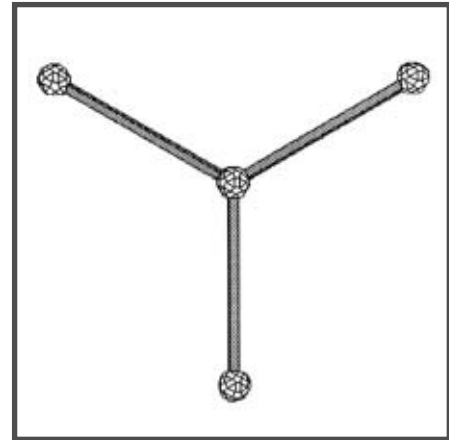
To meet the standard students must build a correctly show the multiple axes of symmetry in the polygons. To exceed the standard they must verbalize a connections between symmetries in geometry and in natural objects.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **number sense and numeration** (NCTM Standard 6).
- * Mathematics standards **addressing geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Follow up this class with a “math hike” in a local park or nature area to search for additional manifestations of numbers in nature. This exploration serves as an introduction to Fibonacci sequences (“Fun Fibonacci”) and other mathematical patterns relevant in biology and geology. Continued work on various types of symmetry (“Rotational Symmetry,” “Translation Symmetries and Tilings,” “Tilings with Multiple Symmetries,” and “Spiral Symmetries”). Discussions on the use of geometry and symmetry in art and design.



Rotational Symmetry

Zome System

Builds Genius!

tribute the Zome System elements. The challenge for each group is to build a more complicated object with the type of symmetry displayed by the latter shapes. Their models should NOT contain reflection symmetry. Allow the students to build for 15 to 20 minutes before choosing a representative from each group to explain the group's design. Pass a mirror around the classroom as they build their designs. *Does the design have another kind of symmetry? What is repeating on this shape? In what way is it repeating?* As they discover the way in which the pattern repeats, ask them what name they would give this kind of symmetry, based on the way it repeats. After discussion, if they have not named rotational symmetry, write it on the board.

Use the mirror to show how an object with reflection symmetry has no "handedness." In other words, it is the same in the reflection as the original. Use the mirror to contrast a rotationally symmetric object, which is *not* the same as its reflection, but comes in a right-handed and left-handed form as described in the picture.

Complete the class with a discussion about different manifestations of rotational symmetry. *Where do we see this kind of symmetry in nature? In art? In the classroom?*

The students should make a drawing of an object with rotational symmetry in their math journals. The drawing should be accompanied by their own definition of this symmetry.

Assessment:

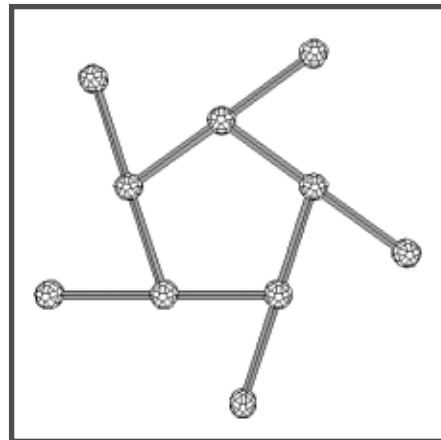
Discuss the definitions offered by the students individually and with the class, and review their math journals. To meet the standard students must build a model and correctly show how a rotation of it gives a repeating symmetry. To exceed the standard they must verbalize a correct definition of rotational symmetry.

Standards Addressed:

* Mathematics standards addressing geometry and spatial sense (NCTM Standard 9).

Transfer Possibilities:

The class is a prerequisite for the following plans on symmetries ("Translation Symmetries and Tilings," "Tilings with Multiple Symmetries," and "Spiral Symmetries"). It is also a good introduction to discussions on rotational symmetry in nature, and its use in art and design.



Mathematics Basic Concept

Lesson Objective:

Students will learn the concept of **perimeter**. For young students, perimeter is not going to be an exact measurement. Understanding the basic concept of perimeter is more important at this point.

Prerequisite Skills:

Knowledge of basic geometric shapes (“Geometric Shapes”).

Time Needed:

One class period of 45-60 minutes.

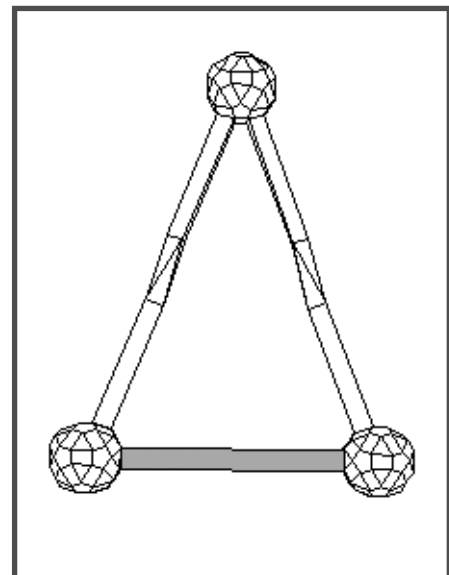
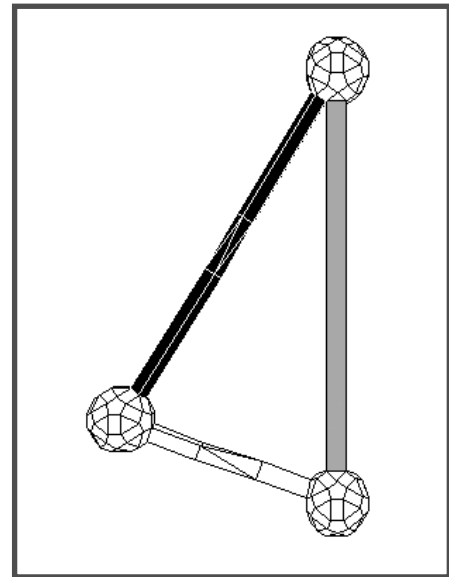
Materials Needed:

- One or two Zome System Creator Kits for 25-30 students.
- String
- One scissors for each team of 3-4 students

Procedure:

Introduce the term perimeter. Explain that the perimeter is a measurement of how far it is around an object. *Why is it useful to know how to measure perimeter? In what kind of situations do we need to know the perimeter* (when building a fence, making clothing, etc.)? *How could we decide the perimeter of a geometric shape like a triangle?* Brainstorm ways determining the perimeter of a triangle. Explain that that it is possible to use string to measure perimeter, and model how this is done using a Zome System triangle.

Divide the class into teams of 3-4 students, and distribute Zome System pieces and other materials. Each team will be responsible for making a designated shape; square, triangle or rectangle. Once the teams have made a few different sizes of their shape, they are to measure their perimeters with the string. Encourage discussion about



What Is Perimeter?

Zome System

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the differences in perimeter between shapes made with long struts and shapes made with medium or short struts. Have each group save one string they used to measure the perimeter of one of their shapes to present to the class.

Bring the class back for presentations of findings, and a large group discussion. Presentations can be made either by a team representative, or jointly. *What did they discover? Which shape has the longest perimeter? Do all triangles have the same perimeter? Do shapes with more struts always have a longer perimeter than those using fewer struts?* Ask the students to compare the strings to one another to see which shape had the largest perimeter and which had the smallest. Pose the question why one shape has a longer perimeter than other.

For younger students, the activity is intended to expose them to the perimeter concept. Older students (i.e., second graders) can write in their math journals about perimeter.

Assessment:

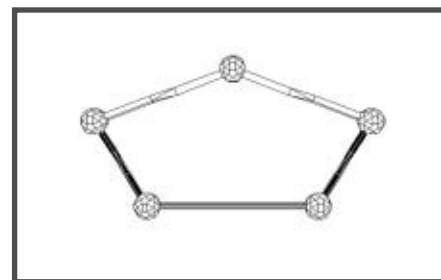
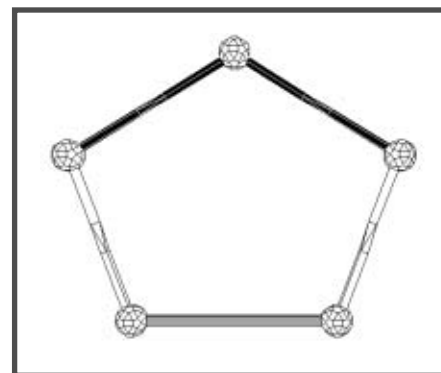
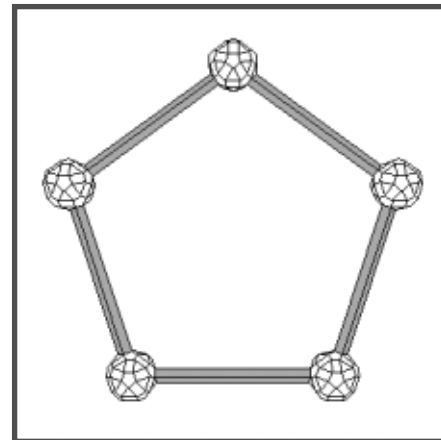
Observe the students while they work in their teams, and question them on their findings. To meet the standard students must build simple polygons and be able to measure their perimeter with the string. To exceed the standard they must point to practical uses of perimeter measurement, and be able to determine which shapes have the longest perimeter.

Standards Addressed:

- * Mathematics standards addressing mathematical connections (NCTM Standard 4).
- * Mathematics standards addressing geometry and spatial sense (NCTM Standard 9).
- * Mathematics standards addressing measurement (NCTM Standard 10).

Transfer Possibilities:

Continued exploration of polygons and 3-dimensional forms (“2-D Polygons,” “Try the Triangle,” and “2-D and 3-D Shapes”). More advanced work on applications for perimeter and other measurements (“Perimeter Puzzles,” “What is Area?” and “Volume for Beginners”).



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Perimeter Puzzles

Mathematics Basic Concept

Lesson Objective:

Students will learn the concept of a perimeter and learn how to measure it in a structure.

Prerequisite Skills:

Knowledge of basic geometric shapes ("Geometric Shapes," "2-D Polygons," and "Try the Triangle,").

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

- Two Zome System Creator kits for 25-30 students.

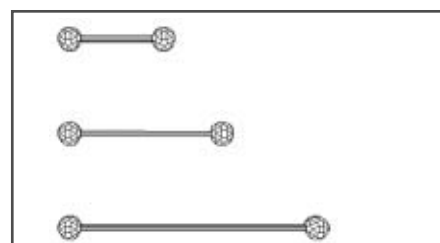
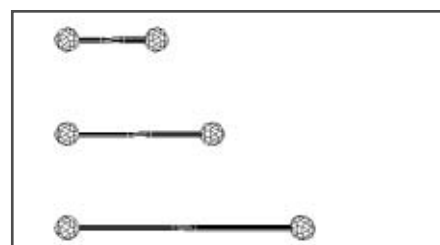
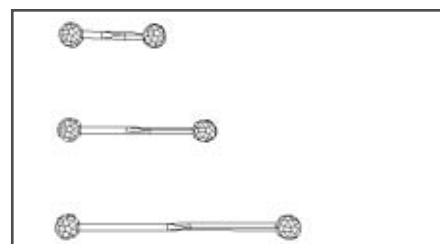
Procedure:

Begin by asking the students how many of them have a fence surrounding their backyard, apartment building, or playground? *What is the purpose of having a fence* (for safety, to keep the dog in a yard, privacy, etc.)? *Who builds fences? How do people go about planning for a fence? How do they know how much materials they will need for their fence?* Allow students to discuss the idea that it is necessary to measure in order to determine the amount of fencing, etc.

Draw a 4 sided figure on the board. Ask students how many sides the figure has. Ask them how they know that the answer is 4. They will response, "by counting". Explain that when someone creates a fence or a boundary around something they *count* the distance. This distance is known as **perimeter**.

The challenge for the class is to calculate the perimeter of various geometric shapes using Zome System. Have students copy the chart in their math journals.

Divide the class into teams of 3-4 students, and distribute the Zome System pieces. The teams are to build the following geometric shapes and then calculate their



<u>Size</u>	<u>Color</u>	<u>Measurement</u>
short	yellow	1
short	red	2
short	blue	3
medium	yellow	4
medium	red	5
medium	blue	6
long	yellow	7
long	red	8
long	blue	9

perimeters; square, pentagon, rhombus, right triangle, equilateral triangle, rectangle. All students should record their team's activities and findings in their math journals.

Finally, each team should be asked to build a fence around one of the desks in the classroom. The fence is to be built it using the most cost effective plan. Students should refer to the chart previously copied into their math journals. The measurement on the chart is equal to the cost of each strut. For example, one small yellow strut costs \$1.00. One long blue strut costs \$9.00. Discuss appropriate strategies before building occurs. *Can we predict how much the fence is going to cost? Which color fence is going to be the cheapest? Is a fence that resembles one of the polygons above going to be cheaper or more expensive than a free-form shape?*

Students should experiment with various approaches. All work should be included in math journals. Conclude the class with a large group discussion of results. The lesson can be extended to include actual measurement of the struts in order to find perimeter.

Assessment:

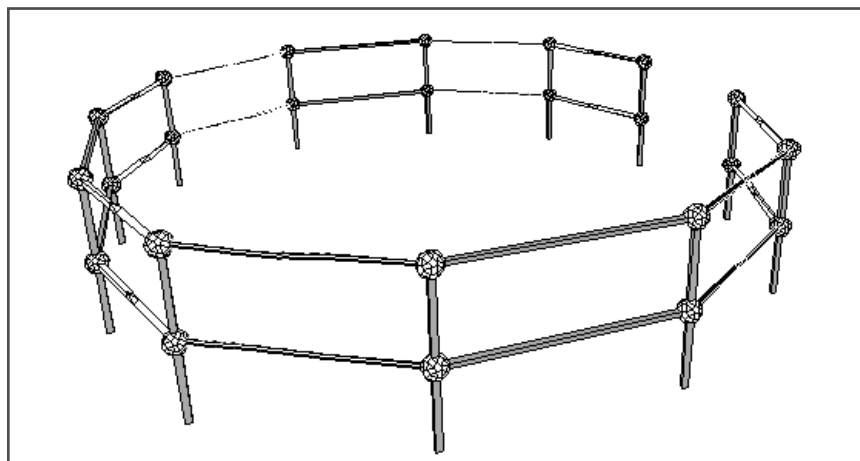
Observe students as they work, and check written work in math journals. To meet the standard students must build the listed polygons and determine their perimeter. To exceed the standard they must complete the team "fence" and define the budget for this construction.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

More advanced work on measurement ("What is Area?" and "Measurement of Space II - Volume") and budgeting for construction projects, and other economic concepts ("Tallest Tower in the World" and "Bridge Building Unit").



Zome System

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What Is Area?

Mathematics Basic Concept

Lesson Objective:

Learning the concept of **area**. For young students, area is not going to be an exact measurement; understanding the general concept of area is more important at this point.

Prerequisite Skills:

Knowledge of basic geometric shapes. If students will build their own shapes, they need to have played with Zome System before.

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

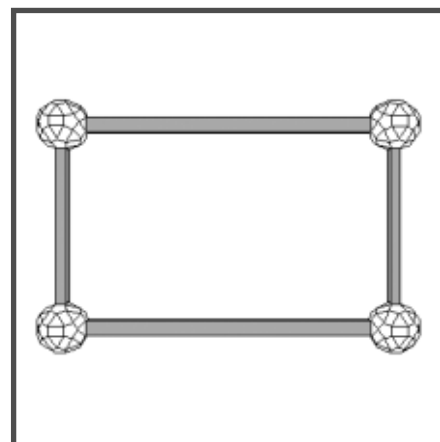
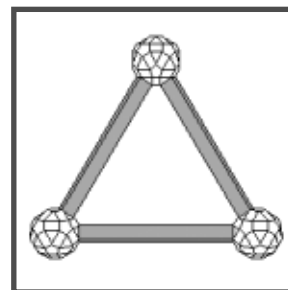
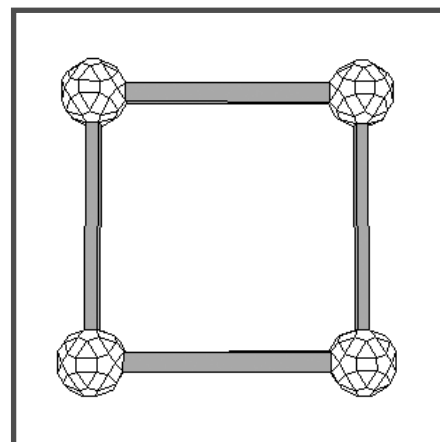
- One or two Zome System Creator Kits for 25-30 students.
- Large bag of dried lima beans, styrofoam peanuts, or similar objects.

Procedure:

Introduce the term **area**. Explain what area of an object means. *How could we find the area of a square? How would you describe the area once you found it? Can anyone think of how we could use these lima beans to find the area of a square?*

Explain to the class that they will use lima beans as the unit to determine the area of certain shapes. Model how to determine the area of a square using the beans (i.e. fill the square shape with lima beans and then count how many beans were used.)

For first and second graders, divide the class into teams - square, rectangle and triangle teams. Have each team build the shapes they will work with. Each team will fill their shapes with beans to determine its area. One student on each team should act as record keeper. He or she should record the number of beans it took to find the area of one



What Is Area?

Zome System

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of their shapes. For kindergartners, the teacher will do this in a large circle with all students. Once the area has been determined for the different shapes, discuss what was learned. *Which shape has the most area? Which shape has the smallest area? Why? Can anyone think of another type of unit that could be used to measure the area of a shape? Why do you think an area rug is called an area rug?*

Assessment:

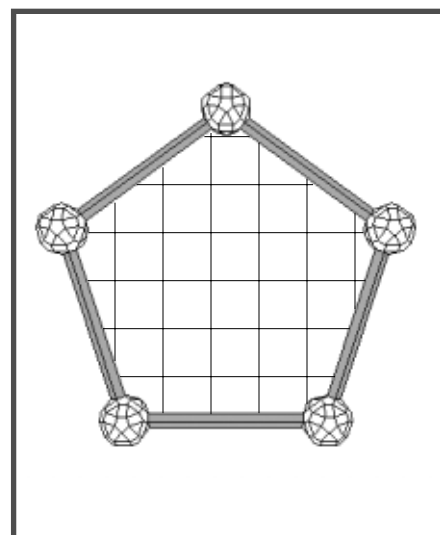
Observe the students while they work in their teams. Question them on their findings. For younger students, the activity is based on exposing them to the area concept. Older students (i.e., second and third graders) can write in their math journals about area. To meet the standard, students will understand the concept of area. To exceed the standard, students will understand that the size of the area can be calculated using numbers and an equation.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

Use of standard measurement units to determine area. Exploration of the concept of volume (“Volume for Beginners”).



Zome System

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Volume for Beginners

Mathematics Basic Concept

Lesson Objective:

Introduction to the concept of volume for younger students, who have not learned multiplication. At this age, volume is not going to be an exact measurement. Exploration and problem solving is more important at this point.

Prerequisite Skills:

Students need to have played with Zome System before, know basic geometric shapes (“Geometric Shapes,” and “Geometry is All Around Us”), and be familiar with concepts of area (“What is Area?”).

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

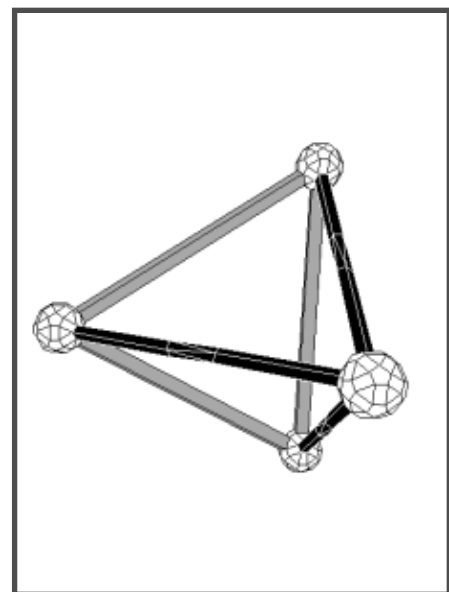
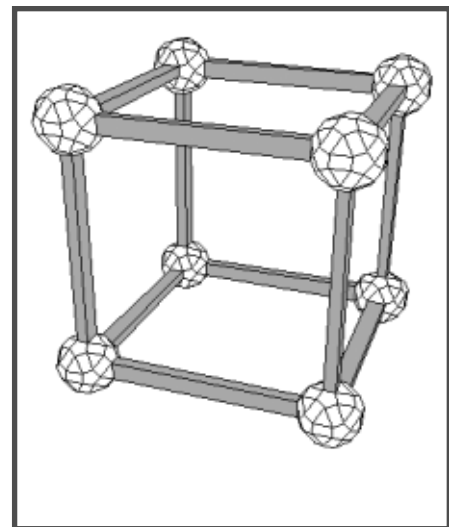
- Two Zome System Creator Kits for 25-30 students
- Large bag of dried lima beans, styrofoam peanuts, or similar objects.

Procedure:

Explain to the students that they are going to start working on something called volume. *Does anyone know what that word means?*

Divide the class into teams of 2-3 students, and distribute the Zome System pieces. The first task for the teams is to build a small “box”. The box can be made up of any of the polygons they already know. Cubes, pyramids, various prisms, etc., are all acceptable. Allow 10 minutes for the teams to agree on the shape of the box, and complete building it.

Now explain that volume is a measurement of how much *can be fitted inside an object*. *How could we find the volume of an object? How would we describe the volume once we find*



Volume for Beginners

Zome System

Builds Genius!

it? Explain to the class that they will use lima beans as the unit to determine the volume of their box. Distribute the beans to the teams. The task for the teams is to figure out a method of measuring how many beans fit into the box. They must take notes of their discussions and different ideas.

Circulate and question the students as they experiment. Some teams are likely to use their hand to stop the beans from running out the sides of their box. Others may establish how many beans fit on the bottom of their structure, and then estimate how many layers would fit on top of each other. They will get an answer by repeated addition of the number of beans in the bottom. Repeated addition is of course the first step towards multiplication. Yet others will lay the structure on its side, and fill the sides with beans. This way they will be able to get an exact count of the layers.

Break off the exploration after 15 minutes, or when all teams have an answer. Each team should introduce their structure to the class, relate how many beans it can contain, and explain how they came to *this answer*.

Discuss what the teams learned. Was the exercise difficult? Why, or why not? Which structure had the largest volume? Which had the smallest? Which method of measuring bean volume was the fastest? Which was the slowest. Which one was the most accurate? Would all the methods have worked if the structures were much bigger? Is there a quicker way to add up all the layers for those who counted layers? Which units of volume do the students know? In which situations do we need to know volumes? Conclude by asking the students to individually write down some observations about volume and its uses.

Assessment:

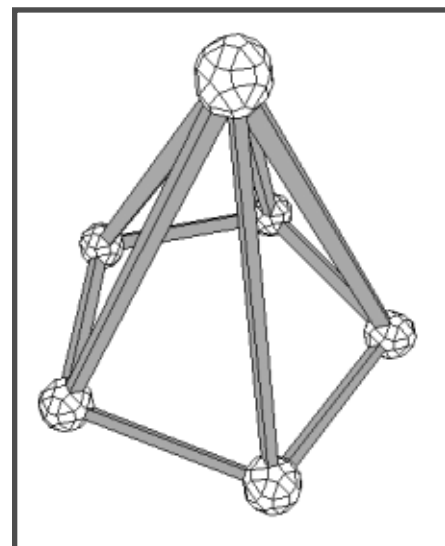
Observe the students while they work in their teams, and question them on their findings. Review team and individual notes about volume. To meet the standard, students must devise a method to calculate the volume (in beans) of a simple structure. To exceed the standard, they must draw some conclusion on the need for a more standardized measuring unit.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

Use of standard measurement units to determine volume. Introduction to multiplication. Study of the interaction between volume and area (“Measures of Space I-Lengths and Areas,” and “Measures of Space II-Volumes”).



Physics Basic Concept

Lesson Objective:

Students start exploring bubble making with Zome System. They will establish that bubbles have 3-fold symmetry.

Prerequisite Skills:

Students should have some background in relating geometric shapes to numbers (“Shape and Number”), as well as the ability to identify a line of symmetry (“What is Reflection Symmetry?”). It is also good if they have experimented with, or discussed, surface tension in the past.

Time Needed:

One class period of 45-60 minutes.

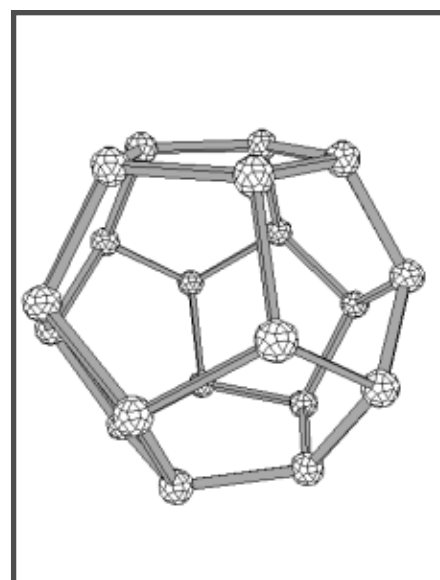
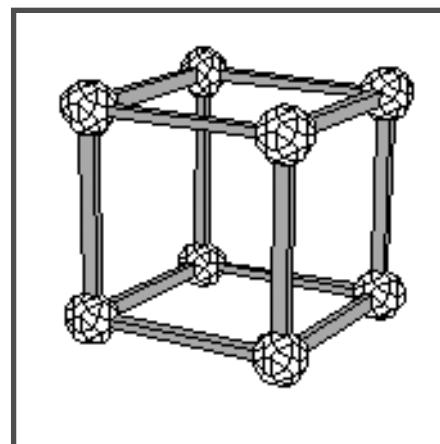
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Five or six buckets of water with approx. 1/3 cup of liquid dish soap in each (Dawn works well)
- A few drinking straws
- Overhead projector

Procedure:

Prepare by reading the section on bubbles (pages 5-7) in Zome System Manual 2.0, the color brochure that came with the Zome System kit. Mix the bubble solution the day before the class for better lasting bubbles. A few drops of glycerin may also be added. Test that the solution is strong enough to make bubbles. Place the buckets in your class room so the students have space to work around them. Place newspapers on the floor and workspace to absorb the drips.

Divide your class into teams of 3-5 students, and assign each team to one of the buckets. Ask the students if they know how soap bubbles work. *Are bubbles always round?*



Beginning Bubbles

Zome System

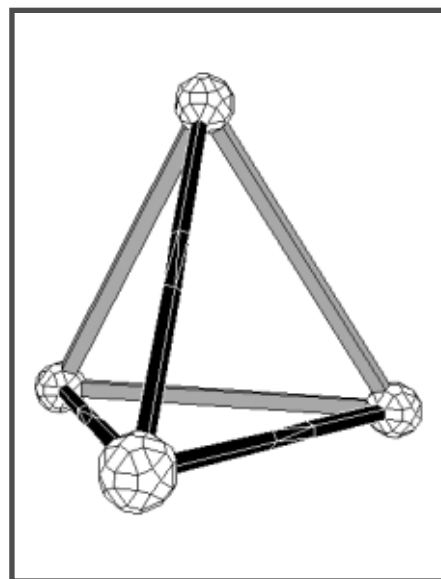
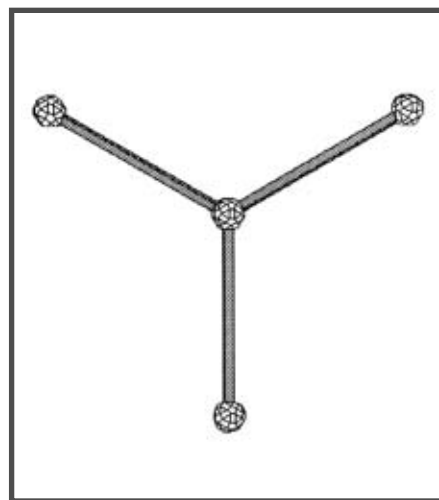
Builds Genius!

How can we find out to be sure? Lead the discussion until the students have established that we must do experiments to learn certain things. Tell the students that they are going to make the bubbles by making Zome System structures and dipping them in the solution. To avoid a big mess, discourage the students from blowing bubbles. During the first 20 minutes they may experiment freely, but must take notes of their findings.

Walk around and assist as needed as the students experiment. Use examples from the students' structures to point out interesting features to the other ones using the same bucket. *Which is best, a flat polygon, or a 3-D structure? Why? Why does a dry finger pop the bubble, while a wet finger does not?* Students will eventually notice that a simple structure such as a cube or a tetrahedron gives cleaner, more interesting, combination of bubble surfaces than a complex piece with a large number of struts. Also, guide the students if they do not realize that they can trap air inside the structures by first dipping an entire structure, and then submerging it only partially into the solution. A "double-dipped" cube will have a cube-shaped bubble suspended inside, and a tetrahedron can contain a tetrahedron-shaped bubble, and so on, with increasingly complex shapes. Change the size of a bubble by blowing in or sucking out air with the aid of a drinking straw (which must be wet, as a dry straw will pop the bubble).

As a whole class, collect students' findings on the board or on flip chart paper. *Why do the bubbles connect and pull in towards the center of the Zome System structures, instead of staying on the outside* (the bubbles are striving for a minimal surface. Younger students will understand the concept of a bubble being "lazy"; it takes less bubble, less energy, to create a shared bubble surface in the center of the structure)? *Did anyone notice how many bubbles can touch each other at the same time?* (Either spherical bubbles in the bucket, or bubble films inside the structures.)

Place a transparency sheet on the projector and pour a very small amount of bubble solution on it. With your straw, blow a number of bubbles on the transparency to show how bubbles always will meet in threes. The shad-



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Beginning Bubbles

ow cast by the projector will show a perfect 3-fold symmetry wherever the bubbles meet. *What is the angle between the bubbles? Can we build a Zome System structure that shows the same 3-fold symmetry?*

Go back to the simple Zome System structures that made the best bubbles earlier. *How many bubble films meet to form the lines in the bubble structures?* The students will find that the most striking bubbles were made by structures with no more than 3 struts in each node. Hold a short discussion about the consistency of the number 3 in the bubbles.

There are several more Zome System classes that use bubbles. The solution can be saved for these classes.

Assessment:

Observe students as they work and questions the groups about their findings. Ask a few students to demonstrate the 3-fold symmetries to others in their group. Older students should write up their findings in their math journals. To meet the standard the students must verbalize which type of structures make bubbles effectively. To exceed the standard they must make a connection between bubbles, the Zome System structures, and the number 3.

Standards Addressed:

* Science standards addressing consistency of natural laws, particularly benchmarks on surface tension and energy use in nature.

Transfer Possibilities:

More advanced work with bubbles (“Bubbles II - Surface Tension & Minimal Surface”). Physics classes dealing with surface tension, and minimum energy use in nature.

Mathematics Basic Concept

Lesson Objective:

Students will learn about rhombi (“diamonds”) in relation to the square, and will review the idea of similarity.

Prerequisite Skills:

Some background on exploring polygons (“Geometric Shapes,” “Shape and Number,” and “Similar Triangles”).

Time Needed:

One class period of 45-60 minutes.

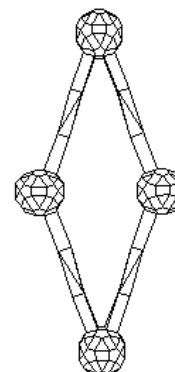
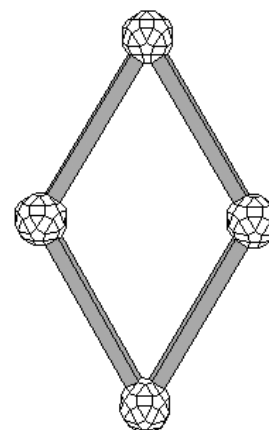
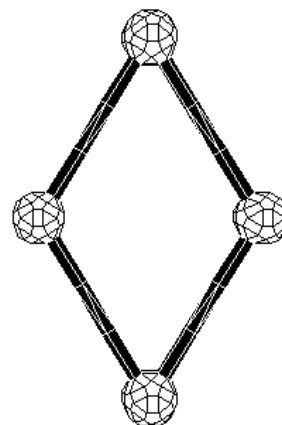
Materials Needed:

- One or two Zome System Creator Kits for class of 25-30 students
- Overhead projector
- Double-sided tape

Procedure:

Prepare for the lesson by cleaning a space on the chalkboard where rhombi built with Zome System can be attached with double-sided tape. The rhombi may be rearranged several time, so it is important that the chalk dust is removed.

Divide the class into teams of 4 students each, and distribute the Zome System elements evenly. The challenge for the teams is to create as many flat rhombus (or diamond) shapes as possible in 20 minutes. They should also find a method of classifying all the rhombi they discover. If students ask what is meant by “rhombus,” encourage them to try to come up with a definition on their own. Suggest that the students build their best guess. They can then compare it with other rhombi being built by their team members. The teams should discuss all the shapes, agree on the definition of a rhombus, and classify them into a system that makes sense to them. Students must



make notes of their findings and definitions in their math journals.

When the work is completed, one representative from one of the teams should present to the class, all the rhombi her/his team discovered. The presenter may use the overhead projector so everyone can see the outline of each rhombus. She/he should also classify the rhombi according to the system agreed to by the team. The teacher or other students should attach the rhombi to the chalkboard using double-sided tape.

Allow a representative for each of the other teams to compare their findings to the display on the board. *Did they find any additional rhombi? Are all the displayed shapes really rhombi? Are there other ways of classifying the rhombi (fattest to skinniest, biggest to smallest, by color)?* Ask teams with “missing” rhombi to bring them up and incorporate them into the classification system. When the class is convinced that all possible rhombi have been classified in the best possible system, they should discuss their findings. *What is the definition of a “rhombus”? Do all rhombi have the same number of struts? Nodes? How many of each? Are they always made of just one color strut? Why, or why not? How many different blue rhombi can be built? Yellow ones? Red ones? What’s the difference between a small red rhombus and a large red rhombus? What’s the same? Is a square a rhombus?*

What do squares have in common with other rhombi? How can you turn a square into a rhombus? Can a square cast a rhombus shadow? Can a rhombus cast a square shadow?

You may wish to introduce new vocabulary. A 2-dimensional figure with four sides of equal length is a **rhombus**. Two rhombi of different sizes with equal angles are **similar**. A **square** is a special type of rhombus with 90° angles. A **kite** has sides of two different lengths. Kites are not rhombi.

Assessment:

Observe students while they work, and review their individual and group notes. To meet the standard, students must define a “rhombus,” and be able to organize these shapes into a consistent classification system. To exceed the standard they must distinguish between rhombi, squares, parallelograms, and kites.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

Expansion into more advanced 2-, and 3-dimensional forms, and their nomenclature (“2-D and 3-D Shapes,” “What are Quadrilaterals?” and “Naming 2-D and 3-D Shapes”). Exploration of the characteristics of specific rhombi (“Non-Periodic Tilings-II: Richert - Penrose Tilings”).

Mathematics Basic Concept

Lesson Objective:

Students will learn to define and build all quadrilaterals, including: squares, rectangles, trapezoids, rhombi, parallelograms, kites, and non-convex quadrilaterals.

Prerequisite Skills:

Knowledge of basic polygons ("2-D Polygons").

Time Needed:

One class period of 45- 60 minutes.

Materials Needed:

- Two Zome System Creator Kits for 25-30 students

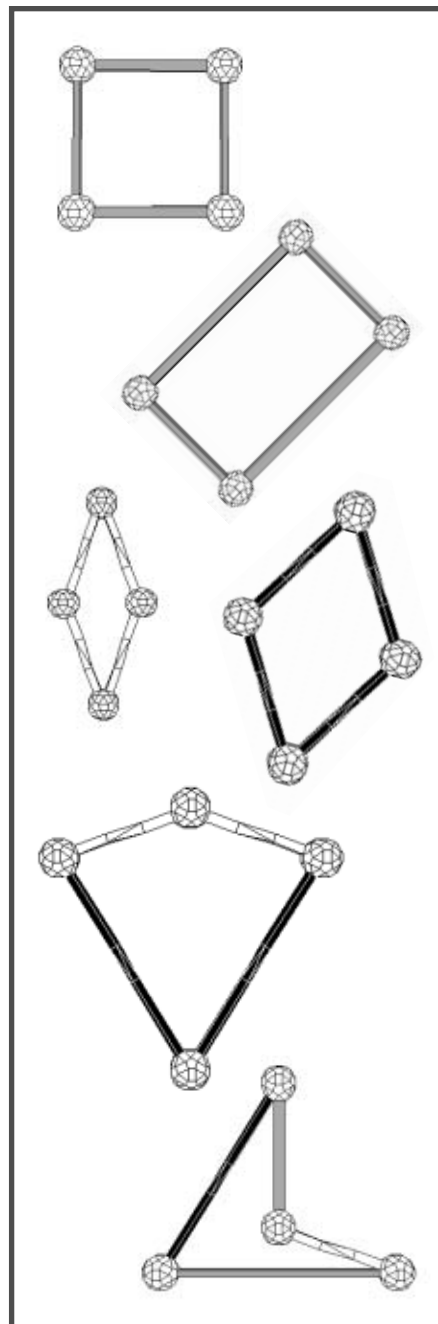
Procedure:

Briefly review the names of the basic polygons your students know. Steer the discussion to the square and the rectangle. *How are these two polygons similar?* Tell the class that any polygon that has 4 sides is called a **quadrilateral**. *Can your students name any other quadrilaterals?*

Divide the class into teams of 3-4 students, and distribute the Zome System elements. The challenge for each team is to build as many different quadrilaterals as possible in a 15 minute period. They should discuss the name of their shapes as they build. Circulate and assist the groups as necessary. *Do the quadrilaterals have to have identical angles? Do they have to have any **parallel lines**?*

Bring the class back together and ask a team representative to demonstrate the shapes they have found. Discuss the shapes as they are presented. Review any concepts the students need to make their definitions, for example: what is a **convex** versus a **non-convex** polygon. *How is each quadrilateral different from the others? What makes it unique?*

Make a table on the chalkboard naming the shapes and



What Are Quadrilaterals?

Zome System

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listing definitions the students agree on. For instance: Quadrilateral: Any polygon with 4 sides; Trapezoid: At least one pair of parallel sides. All 4 sides can be of different length; Parallelogram: Two pairs of opposite sides that are parallel to each other. Opposite sides are of equal lengths; Rhombus: Two pairs of opposite sides parallel. All 4 sides are the same lengths; Square: Opposite sides are parallel. All sides are of equal length. All angles are 90° ; Rectangle: Opposite sides are parallel. Opposite sides are of equal length. All angles are 90° ; Kite: No parallel lines. Opposite angles are identical.

Draw the “family chart” outlined on this page on the board and discuss different quadrilaterals relate to each other. *Are all squares rhombi? Are all rhombi squares? Are all rectangles parallelograms? Are all parallelograms rectangles? Are all parallelograms trapezoids? Are all trapezoids parallelograms?* Continue the questioning until the students can distinguish between the various polygons.

The students should write the agreed upon definitions, and draw examples of the different quadrilaterals, in their math journals.

Assessment:

Observe students as they work, and take notes during their discussions. Review definitions and drawings in math journals. To meet the standard students must name the various quadrilaterals correctly. To exceed the standard they must also verbalize definitions that differentiate the quadrilaterals.

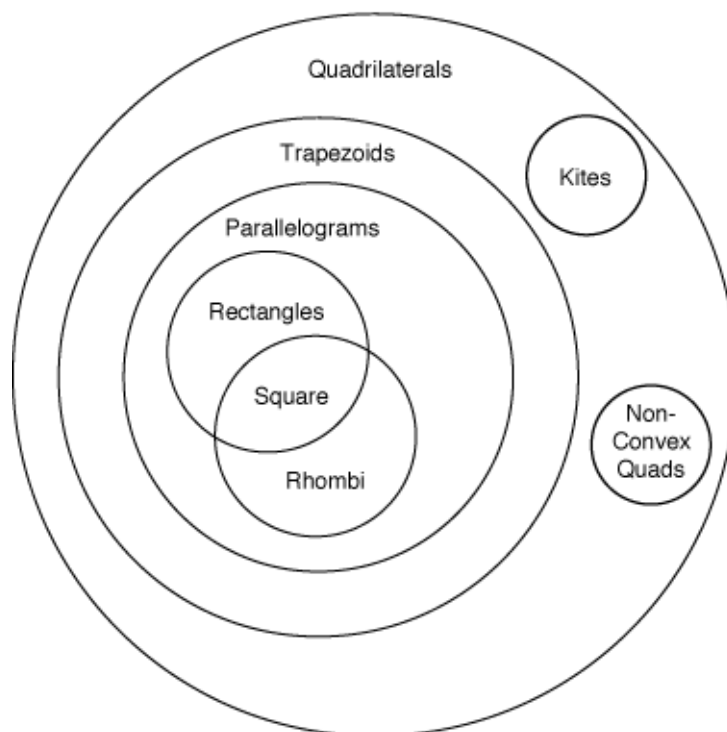
Standards Addressed:

* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Work with tilings (“Tiling with Quadrilaterals”, “Symmetries in Quadrilateral Tilings,” and “Non-Periodic Tilings-II: Richert-Penrose Tilings”).

Families of 4 Sided Polygons



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Mathematics / Art Basic Concept

Lesson Objective:

Students will learn the basic concept of tessellation and repeating patterns.

Prerequisite Skills:

Knowledge of basic geometric shapes.

Time Needed:

Two to three class periods of 45-60 minutes.

Materials Needed:

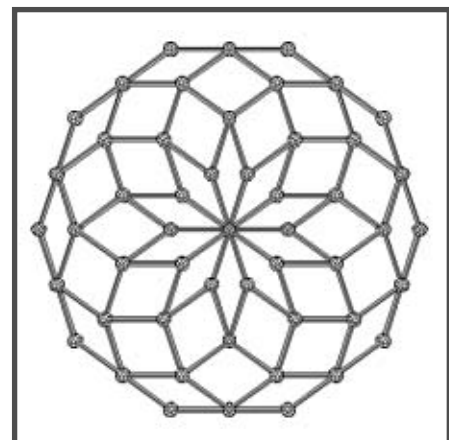
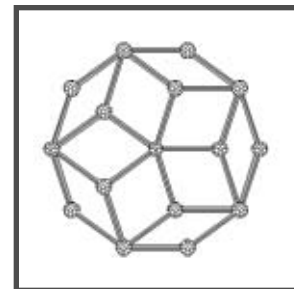
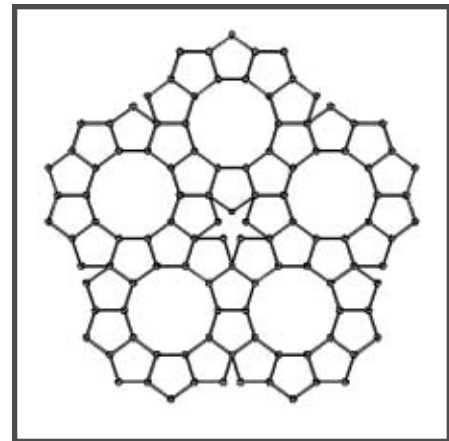
- Two Zome System Creator Kits for 25 - 30 students
- Eight Hands Round by Ann Whitford Paul
- Overhead projector if available

Procedure:

Start the class with a brief review of the concepts of shapes, angles, triangles, and cubes. As a group, brainstorm all the different shapes they can remember.

Then, read the book to the class. As you read, have students notice the various geometric shapes on each page. *What do you notice about the patterns on the quilts? Which geometric shapes are used? Is there just one shape used in the quilt, or are there combinations of several shapes?* Introduce the term **tessellation** and have students look around the room for examples (i.e. ceiling tiles, calendars squares, etc.)

Tell students that they have been commissioned by the US Commission for Modern Art to make a unique patchwork quilt. A transparency of a "letter" from the Commission can be shown on the overhead. Unlike the quilts in the book, which are made from material, these quilts will be made out of Zome System. The quilt should have a repeating pattern using geometric designs. All



quilts must be assembled for public viewing by a specified deadline. Require the quilts to a certain size (i.e. 3 ft x 3 ft.). The teams will receive an imaginary salary based on cooperation, problem solving, and completing the piece on time. The salary will be decreased if the students fail to meet the above expectations.

After reading the letter as a class, your students should discuss how they might go about planning for this project. Some teams may decide to plan their quilt on paper before beginning with the Zome System, while others may use geoboards.

Once the planning is completed each team should be assigned a large flat surface. A section of the class room floor is appropriate. Remember that the quilts will have to be stored between class periods. Students should be periodically reminded about their due date as they work on their project.

Once quilts are completed, the class can have a special viewing party. The atmosphere of simulate a fancy art exhibit can be invoked by serving popcorn or other snacks and playing a tape of classical music. Each group should be asked to present their creation. *Which geometric shapes did they use? How did they arrive at their final quilt? Did they learn what shapes tessellate on their own, and which must be combined with other shapes?*

The teams should also write about the experience in their math journals, and possibly compose a letter to the Commission discussing their final product. The letter should include a picture and a description of their piece. Likewise, they can reflect on their group skills and discuss if they were deserving of the full project salary.

Assessment:

Observe students as they work, and question them about their designs and what they are learning about tessellation. Review their math journals, and their reports to the Commission. To meet the standard the students must complete a 3' X 3' Zome System tiling, and be able to describe which geometric shapes in contains. To exceed the standard they must also attempt to formulate a general rule about which geometric shapes tessellate on their own, and which ones do not.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Language standards addressing **descriptive writing**.

Transfer Possibilities:

Exploration of more advanced tessellations (“Plane Patterns”, “Kepler Tilings”, and “Richert-Penrose Tilings”). Use of tessellations and mosaics in art and design.

Mathematics / Art Basic Concept

Lesson Objective:

Students will understand how any quadrilateral can periodically tile a plane out to infinity (indefinitely) without gaps. Students should try to determine how this is possible using a proof based on angles.

Prerequisite Skills:

Knowledge of basic polygons ("2-D Polygons," "What are Quadrilaterals").

Time Needed:

One or two class periods of 45-60 minutes

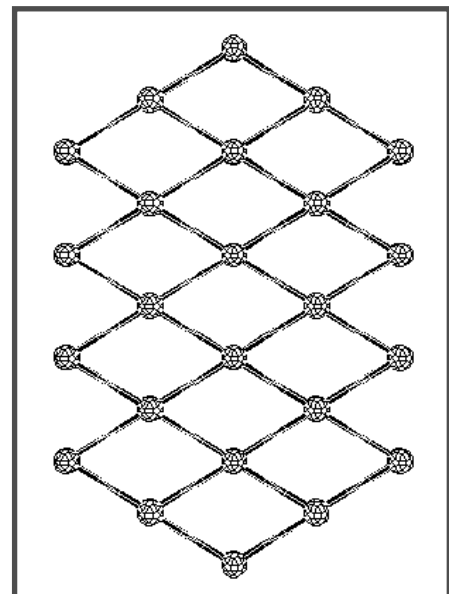
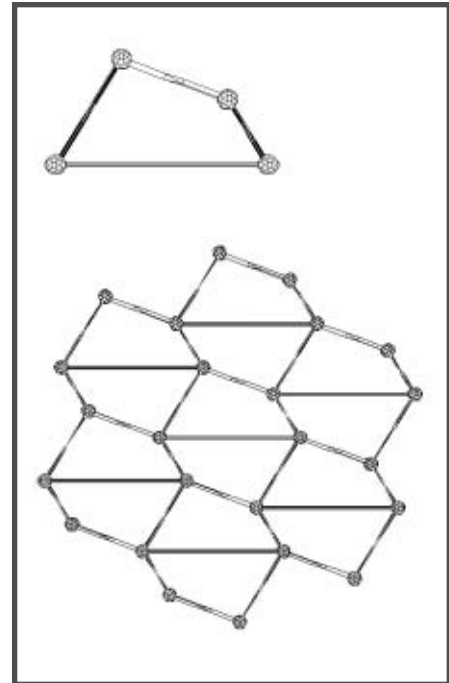
Materials Needed:

- Two Zome System Creator kits for 25-30 students
- One sheet of construction paper per team of 3-4 students
- One pair of scissors per team

Procedure:

Start with a brief discussion on the concept of **tiling**. *Where do we find tilings? Why is it used? Are there any artistic applications of tilings? Can we find examples of tiling in everyday life?* Review the various quadrilaterals the students defined in the "What are Quadrilaterals?" lesson. *What were the eight variations of quadrilaterals? Can any of the quadrilaterals be used to tile a plane? Which four sided shapes can tile and which can not? Are there four sided shapes that can tile more than one way?*

Divide the students into work teams and distribute the Zome System elements. Let each team choose one or two examples of the eight types of quadrilaterals. The challenge for each team is to determine whether their shape will tile or not, by building a tiling in Zome System. The



Tilings with Quadrilaterals

Zome System

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students should be allowed to build their tiling as large as they can in a 20 minute period. During this time, circulate and assist the students as necessary. *Does the shape need to be inverted or flipped over?*

After the tilings are built, ask the students what they found. *Were there any four sided shapes that would not create a tiling? How can we show that any four sided figure will tile?* One way to experimentally show this is to fold a sheet of paper several times so that it becomes several sheets thick. Cut from this stack a random quadrilateral. This will produce several quadrilaterals from four cuts. Playing with the tiles, the students will quickly see how any quadrilateral will tile. This can also show how all four angles of any quadrilateral will always add up to 360° .

The tilings can be saved for the “Symmetries in Quadrilateral Tilings” lesson.

Assessment:

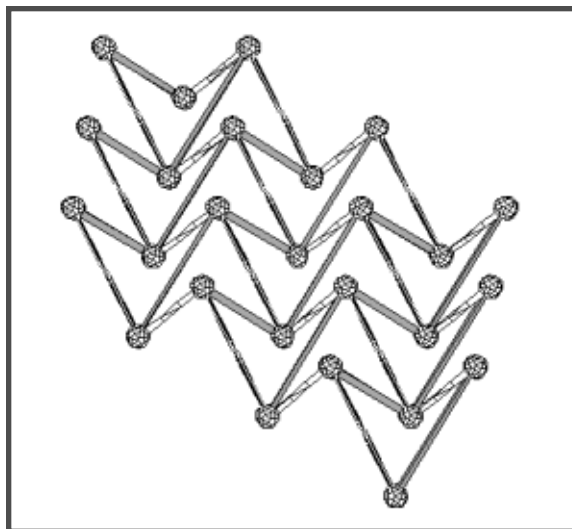
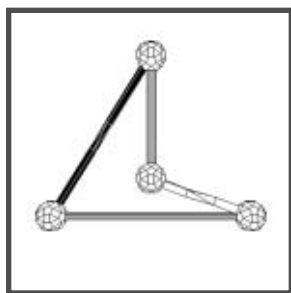
Observe and listen to students as they build their structures. Review notes in math journals. To meet the standard students must create a tiling of quadrilaterals in one plane without gaps. To exceed the standard they must write a definition that relates the tiling properties of quadrilaterals to their combined angles of 360° .

Standards Addressed:

* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

More work with tilings (“Symmetries in Quadrilateral Tilings,” and “Non-Periodic Tilings-II: Richert-Penrose Tilings”).



Mathematics Basic Concept

Lesson Objective:

Students will learn simple building economics and the concept of translation symmetry.

Prerequisite skills:

Familiarity with the concept of planes (“2-D Polygons”), and knowledge of tilings (“Trying Tessellation”, and “Plane Patterns”).

Time Needed:

One class periods of 45-60 minutes.

Materials Needed:

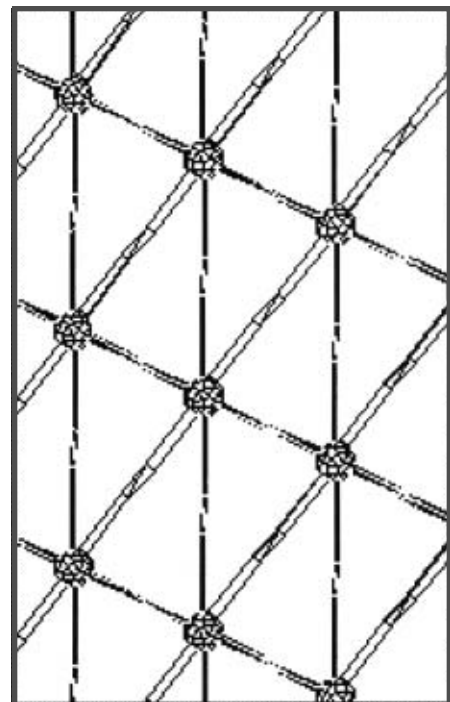
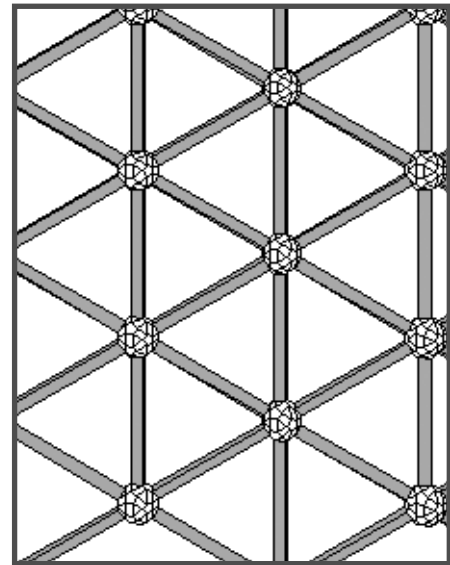
- One or two Zome System Creator Kits for 25-30 students

Procedure:

Divide your class into teams of 4 students each and distribute Zome System components evenly among the teams. In this exploration, it’s important that each team starts with the same number of each Zome System component. Set aside any remainders.

Based on their earlier work with triangles (“Similar Triangles”) and of tilings (“Trying Tessellation” and “Plane Patterns”), challenge each team to build a “flat pattern” based on one unique Zome System triangle repeated many times.

Allow 10-15 minutes for this exploration, during which time you can offer guidance to teams an individual basis. If teams are not sure how to begin, you may can suggest that they first agree on a “seed” triangle. They should then make a few exact copies of the seed triangle, and experiment on how their triangles could together to form a repeating pattern.



Triangle Tiles - I

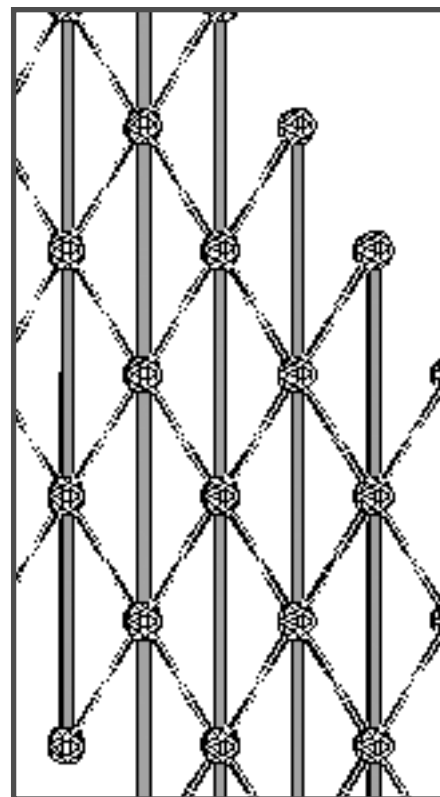
Zome System

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At this point, expand the challenge: “Which team can build the pattern containing the largest number of triangles?” To add an economic dimension to the challenge, mention that inter-team trading of Zome System parts is OK.

When the allotted building time expires, the class should discuss the “market” aspect of the exploration. *Did any team run out of parts? Which kind? Why did (or didn't) they trade with other teams? Did one team offer a better “price” for parts than another? Were some parts more valuable than others? Why? What was the “exchange rate” between blue and yellow struts? Blues and reds? Yellows and reds? Why? Which would result in more total triangle “production:” if all teams made the exact same kind of triangle or if each team produced a different type of triangle? Could you go on building the pattern forever? In “real life,” do all “teams” start out with the “same number of parts?” Why do people use money instead of just trading one kind of thing for another?*

The discussion will lead to a comparison with money transactions. Parallels can be drawn with your students' money experiences, such as how they get money, how they spend it, how they base their economic decisions, etc. It can also lead in philosophical or moral directions, depending on your comfort level. Returning to math, it may be interesting to have your class agree upon and write up a “price list” for Zome System components based on this exploration. Each team needs to save their completed pattern structures for the “Triangle Tiles - II” session.



Assessment:

Observe the building work of each team, as well as the interaction between teams. Take notes during the group discussion. To meet the standard students must build a continuous tiling of one triangle shape. To exceed the standard they must make observations about the perceived value of different Zome System part.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Exploration of more advanced tessellations (“Kepler Tilings”, and “Richert-Penrose Tilings”). Use of tessellations and mosaics in art and design. The lesson is also a useful preparation for sessions on space frames and other architectural structures (“Tallest Tower in the World”, “Bridge Building Unit”, and 3-D Triangle Tiles”).

Zome System

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Translational Symmetry in Tilings

Mathematics Basic Concept

Lesson Objective:

Students will understand how **periodic** tilings always have **translational symmetry**.

Prerequisite Skills:

Knowledge of various types of symmetry (“What is Reflection Symmetry,” “Multiple Reflection Symmetry,” and “Rotational Symmetry”).

Time Needed:

One or two class periods of 45-60 minutes.

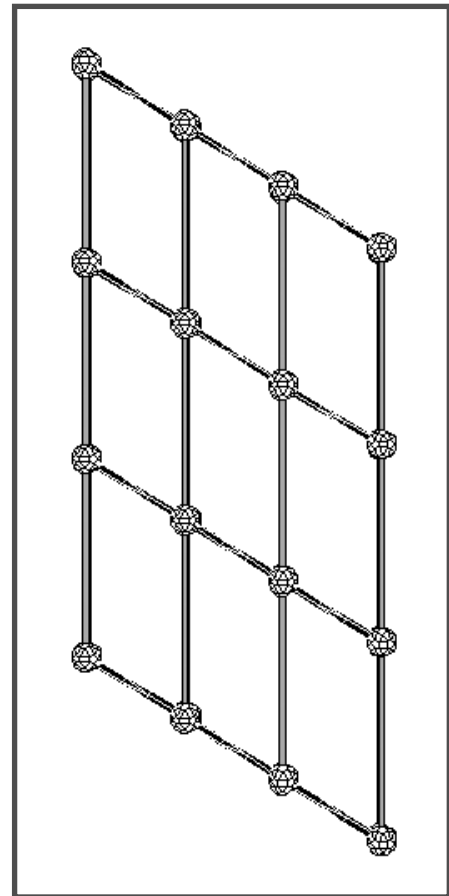
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- A hand-held mirror

Procedure:

Explain to the students that they will continue to discuss further concepts of symmetry. Review reflection and rotational symmetry. *What are other ways that a shape or pattern can repeat itself?* List the students’ descriptions and ideas on the board. If they have not listed or described translational symmetry, let them know that there is another form in which a basic unit can be repeated in an orderly way, but that it is their task to determine what it is.

Divide the class into teams of four to five students and distribute the Zome System elements evenly. Ask each team to build either a parallelogram or hexagon to work with. Allow the teams to build a flat tiling with copies of their shape. *What kinds of symmetry can you find? Does it have rotational symmetry? Where? Reflection symmetry? How? Where? What else is happening?* (The tile moves in a fixed direction over a fixed distance an infinite number of times.) After they have defined this symmetry, tell them it is called translational symmetry. *Where in the “real*



Translational Symmetry in Tilings

Zome System

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*world” do we find translational symmetry (examples include floor tilings, brick walls, carpet patterns, honeycombs, etc.)? Each student should write down his or her own definition of translational symmetry in their math journals. If there is time, the teams can start experimenting with more advanced translational symmetries. Can a translational symmetry take place in 3 dimensions? Which types of shapes can be used in a 3-dimensional tiling without leaving gaps between the cells? Can the shapes fit together in more than one way? Allow the teams to explore how a 3-dimensional lattice can be constructed. As they finish they can be allowed to present their models to the class and show how they show translational symmetry. Do we know any examples of 3-dimensional translational symmetry in nature or the man-made world? Examples that are useful for discussion include the formation of **crystals** when atoms are arranged in this type of symmetrical pattern. Translational patterns are also found in plant cells, honeycombs, and architectural **space frames**.*

Assessment:

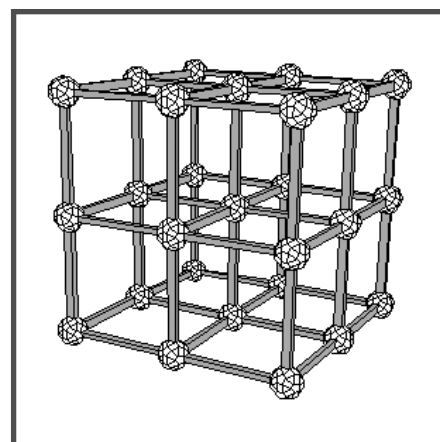
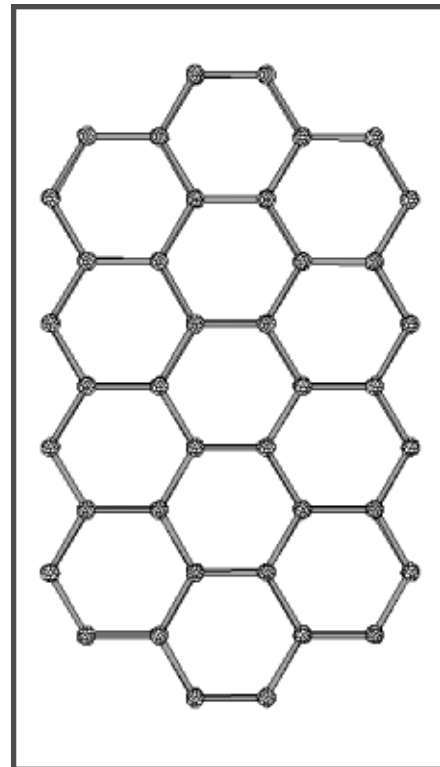
Discuss the definitions offered by the students individually and with the class, and review their math journals. To meet the standard students must build a model with translational symmetry, and be able to show how their pattern repeats itself. To exceed the standard they must verbalize a correct definition of translational symmetry.

Standards Addressed:

* Mathematics standards addressing geometry and spatial sense (NCTM Standard 9).

Transfer Possibilities:

Additional work on symmetry (“Tilings with Multiple Symmetries” and “Spiral Symmetries”). It is also a good introduction to discussions on the use of tilings in fine art and design.



Zome System

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Spinners

Physics / Mathematics Basic Concept

Lesson Objective:

Students will use symmetry to learn about balance, center of gravity and centripetal acceleration.

Prerequisite Skills:

Basic understanding of symmetry and numbers in geometry (“Shape and Number,” “What is Reflection Symmetry?” “Multiple Reflection Symmetry,” and “Rotation Symmetry”).

Time Needed:

One or two class periods of 45-60 minutes.

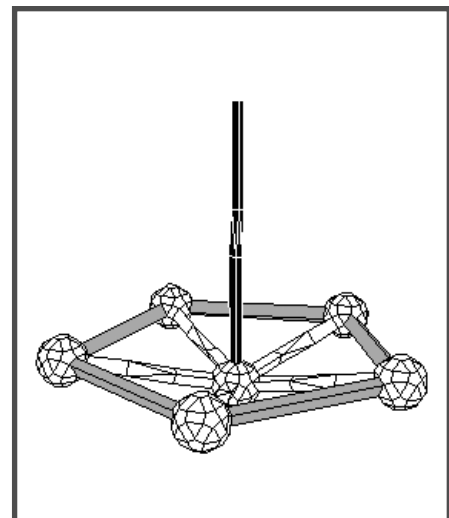
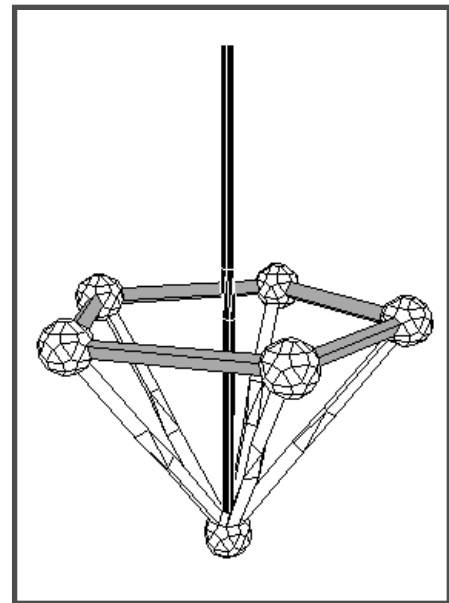
Materials Needed:

- One or two Zome System Creator Kits for a class of 25-30 students
- Stopwatch or timer calibrated in seconds

Procedure:

Divide the class into teams of 4 students, and distribute the Zome System elements evenly. The challenge for the teams is to build the “spinner” which can rotate longest without falling over. Demonstrate with a small spinner (as shown in graphic). While this small model will probably only stay spinning for a few seconds, a larger, more intricate spinners can stay spinning for up to 30 seconds. It’s best to use a smooth wooden or formica surface for the spinner time trials because these surfaces offer less drag and don’t wear on the Zome System nodes. Let the teams know that inter-team trading of parts is allowed.

Circulate and assist the teams as necessary while they experiment. Models can be spun only if one node touches the table and so balance is very important. Every spinner needs a “rotor,” the strut along the symmetry axis of the spinner, which is used to set the model in motion. The



teams are likely to discover on their own that a low center of gravity helps models to balance and spin easier. Ask a few leading questions if a team becomes especially frustrated with a design that isn't working. The teams should make careful notes of their findings as they experiment.

When each team has finished building and testing their spinner, begin the time trials. The teams should introduce their model and explain which of its features will help it spin longer. Each team should be allowed the best of 4 times; that way, each team member gets a chance to set the spinner in motion. Duration is from the time the team member lets go of the spinner until the time a second node touches the table top. It is probably best for the teacher to operate the stopwatch during the time trials.

When a winner has been declared, lead a discussion of student discoveries: *Why did the winning spinner win? Which tend to spin longer, small spinners or large ones? Why? Do larger spinners go faster or slower than smaller ones? Why? Can a spinner be balanced on one node? Is it easier to keep the spinner balanced when it's spinning or when it's at rest? Why? Is there a relationship between the shape of a spinner and the shape of the "rotor" strut that's used to spin it? What do all spinners have in common? What are some differences between various spinners built during this exploration? What are examples of every day spinner-like objects (windmills, propellers, waterwheels, etc.)?*

Items the students may point out include that a low center of gravity gives more stability and therefore longer spin time. A model that is not symmetrical will not spin. Expand the discussion to include the forces involved in a rotation. Compare this to a discus or hammer thrower at the Olympics. *Which forces are acting on the Zome System spinner? Which force is preventing the outer parts of the spinner from flying away like a discus? Why does the spinner eventually slow down and topple over?*

Assessment:

Observe the teams as they experiment, take notes of their discussion, and review their notes in their math/science journal. To meet the standard students must understand the importance of a symmetrical structure with a low center of gravity. To exceed the standard they must attempt to explain which forces are involved in the rotation of the model, and where the energy has gone when the spinning stops.

Standards Addressed:

- * Physical Science standards requiring students to **know and understand common properties, forms, and changes in matter and energy.**
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Mathematics standards addressing **measurement** (NCTM Standard 10).

Transfer Possibilities:

This lesson is a good lead-in to other physics experiments of forces and accelerations. Also good for explorations of gravity forces and stresses ("Tallest Tower in the World," and "Bridge Building Unit")

Zome System

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Printing with Zome System

Art / Mathematics Basic Concept

Lesson Objective:

The students will explore how a shape can be transferred from a Zome System model to paper. The activity reinforces the ideas of vertex points and sides.

Prerequisite Skills:

Students need to have played with Zome System before, and have some knowledge of polygons (“Geometric Shapes”).

Time Needed:

One or two class periods of 45-60 minutes.

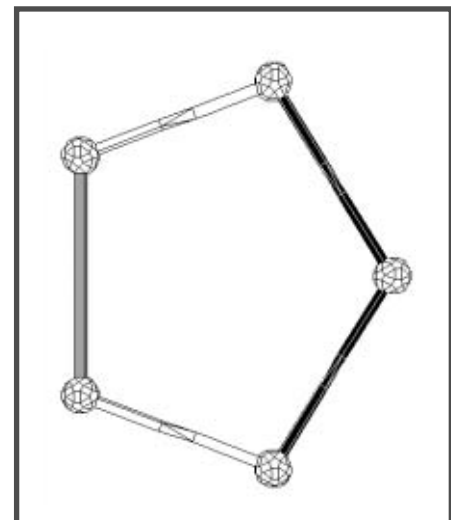
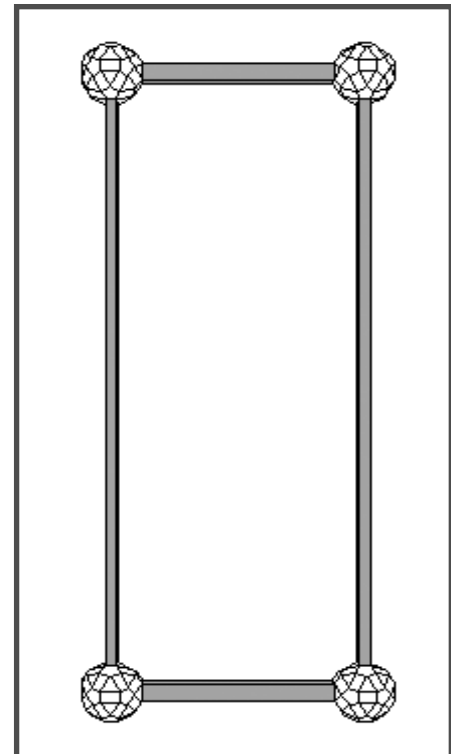
Materials Needed:

- One or two Zome System Creator Kits for 25-30 students
- Trays of pre-mixed, water-based paint
- One sheet of large construction paper per student

Procedure:

Prepare for the class by mixing the paint, and pouring it on styrofoam meat trays or paper plates. The paint should be on the runny side, as too thick paint may take longer to dry. A small amount of liquid soap can be added to the paint, as this aids the clean up. A nice effect is achieved if a different color paint is used for each of the shapes students will be printing with. Now build a number of Zome System polygons; squares, triangles, rectangles, and pentagons. The shapes should be constructed with either small or medium length struts to ensure that their imprints will fit on the particular paper being used. The activity can be planned to start before lunch or recess to allow prints to dry before the activity continues.

Divide the class into teams of 3-4 students. Distribute paint, paper, and a set of polygons to each team.



Printing with Zome System

Zome System

Builds Genius!

Alternatively, students can build the models as an introductory exercise. Explain to the students that each of them will be dipping the shapes in paint and make imprints with it on the paper. The shapes should be placed flat in the tray so each node gets a small amount of paint on one side. Before starting, each group must establish rules on how they will work together. They must decide which direction the shapes will be passed around the table. In addition they must agree which shapes gets dipped in each color. Once a shape is dipped into a color, it remains that color for the entire group.

Each student should stamp each shape once on his or her paper. It is best if the shapes do not overlap. Circulate in the room and assist as necessary.

Once the paint has dried, students should “connect the dots” with a pencil and a ruler, drawing each shape. Finally they should label their printed polygons.

Materials can be set up at a center for later use. At this time students might enjoy making patterns or designs with the different shapes. If students place the shapes in soapy water to soak for a few minutes, clean-up is easy. When the Zome System pieces have been rinsed off they can be placed on paper towels to dry. Store mixed paint in air-tight containers for the “Printing Cubes and Pyramids” lesson.

Assessment:

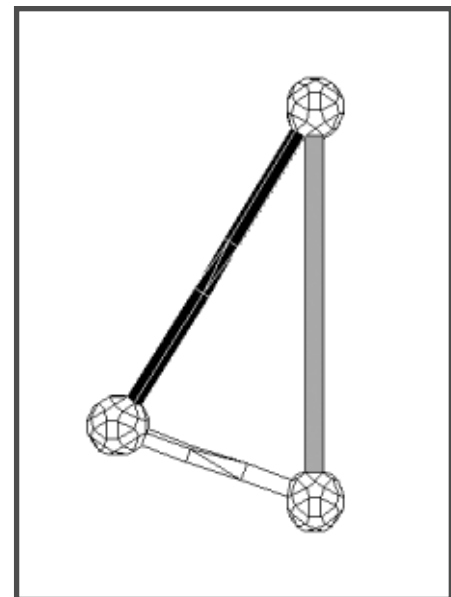
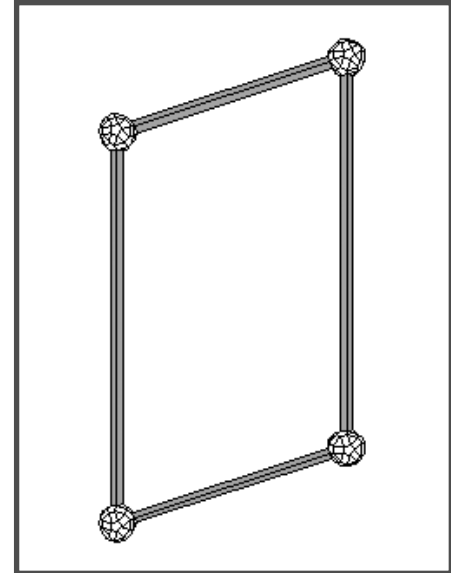
Observe students while they work, and question them individually and in groups to ensure that they can identify the different polygons by counting the sides of each shape. To meet the standard students must print and complete four polygons and name them correctly.

Standards Addressed:

- * Fine Arts standards that **identify and apply the elements of art in a variety of media.**
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Expansion into more advanced printing (“Printing Cubes and Pyramids”) and other means of transferring patterns (“Triangle Tiles - II,” “Cubes - III”).



Zome System

Builds Genius!

Printing Cubes and Pyramids

Art / Mathematics Basic Concept

Lesson Objective:

Students will learn how 2-dimensional shapes can be combined to form 3-dimensional structures.

Prerequisite Skills:

Students need to have played with Zome System before, and have some knowledge of polygons (“Geometric Shapes”).

Time Needed:

One or two class periods of 45-60 minutes.

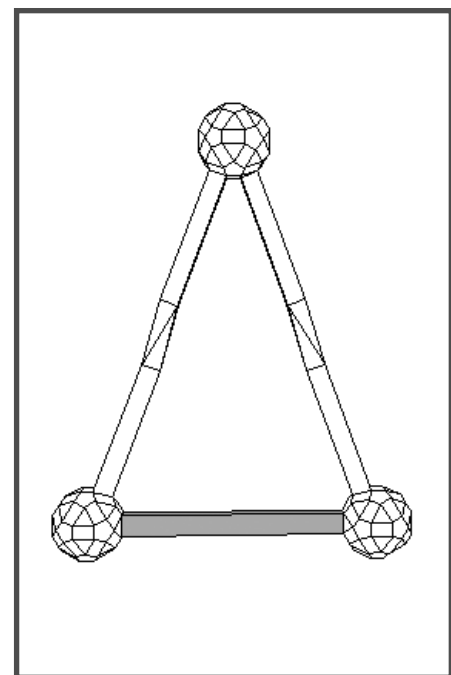
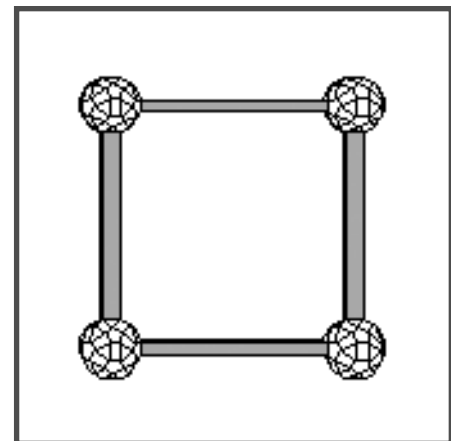
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Trays of pre-mixed, water-based, paint
- Two sheets of large construction paper per student
- A pair of scissors per team of students
- Glue or scotch tape

Procedure:

Prepare for the class by mixing the paint, and pouring it on styrofoam meat trays or paper plates. The paint should be on the runny side, as too thick paint may take longer to dry. A small amount of liquid soap can be added to the paint, as this aids the clean up. Now build one Zome System square and an isosceles or equilateral triangle per student, using short or medium struts. The triangle should have a blue base, of the same length as the side of the square. The activity can be planned to start before lunch or recess to allow prints to dry before the activity continues.

Divide the class into teams of 3-5 students. Distribute paint, paper, and polygons to each team. Alternatively, students can build the models as an introductory exercise. Explain to the students that each of them will be dipping



Printing Cubes and Pyramids

Zome System

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the shapes in paint and make imprints with it on the paper. The shapes should be placed flat in the tray so each node gets a small amount of paint on one side. To help students visualize how they will be printing, ask them to hold the shapes up next to each other in front of their eyes. Overlap one side of the square with a blue side of the triangle. When the students later print the two shapes, the dots from the blue side of the triangle will be placed on top of the dots from the side of the square.

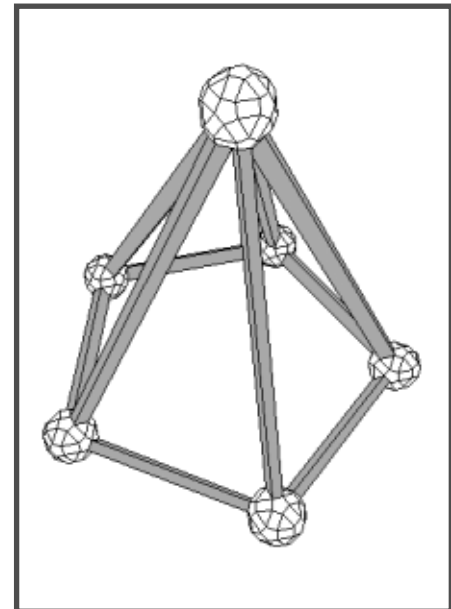
Students begin by printing a square in the center of one paper. Then, carefully overlapping the dots, they print a triangle connecting to one side of the square. They then rotate the triangle to another edge of the square and repeat until they have stamped four triangles.

While the first print dries, students can start on a second piece of paper. Again, they print a square in the center of the paper. This time, however, they print four squares around the center one, each time sharing one common side. Finally they choose one outer square and print one more square along the edge opposite the center square. Allow this to dry.

Instruct students to return to the paper with the square and triangles, and carefully connect the dots with a pencil and ruler. The students then cut out the resulting shape (following only the outer edges). The shape is folded along the original lines of the square. *Which 3-D shape will result when the shape is folded* (a pyramid)? Explain to students that the square forms the base of the pyramid. The paper pyramid can be taped or glued to maintain its shape.

The students should now return to the second paper. *Which 3-D shape will result when this paper is cut out and folded* (a cube)? Repeat the connecting of the dots, cutting, folding, and taping.

Student who finish early can be asked to think of pyramid forms with different bases. They can also build the pyramid, tetrahedron, and cube with Zome System. Close the activity with a discussion about the shapes used, and a review of the new words introduced in the class; **edge**, **face**, **base**, **point** (vertex).



Zome System

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Printing Cubes and Pyramids

Assessment:

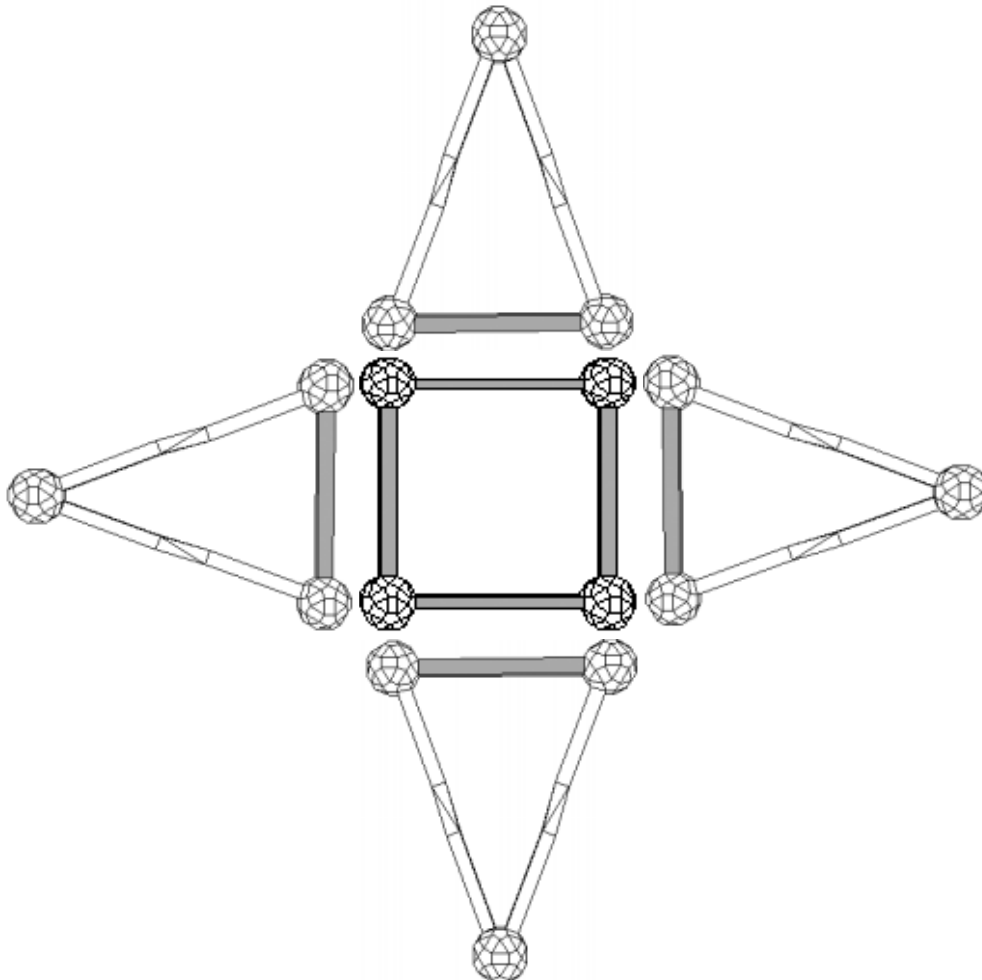
Observe students while they work. Question them individually and in groups to ensure that they understand how the 2-dimensional shapes change into 3 dimensions. To meet the standard students must print, cut out, and fold paper to form the pyramid and the cube. To exceed the standard they must predict another 3-D shape and that can be created by folding paper.

Standards Addressed:

- * Fine Arts standards that **identify and apply the elements of art in a variety of media.**
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Expansion into other means of transferring patterns (“Triangle Tiles-2,” “Cubes 3A”).



Zome System

Builds Genius!

Art / Mathematics Basic Concept

Lesson Objective:

Students will become familiar with **Cubism** as a movement in modern art, and draw parallels between art and mathematics. They will also be able to define a structure's vertices, edges, and faces.

Prerequisite Skills:

Knowledge of basic geometric shapes ("Geometric Shapes," "2-D Polygons," and "Try the Triangle,"). Previous exposure to the use of geometry in art ("Printing with Zome System," "Printing Cubes and Pyramids," and "Trying Tessellation").

Time Needed:

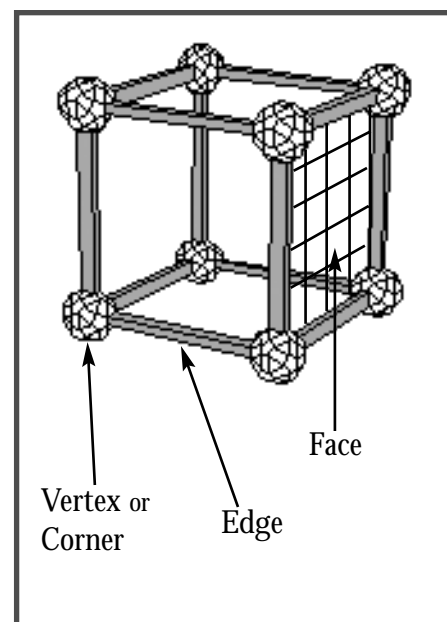
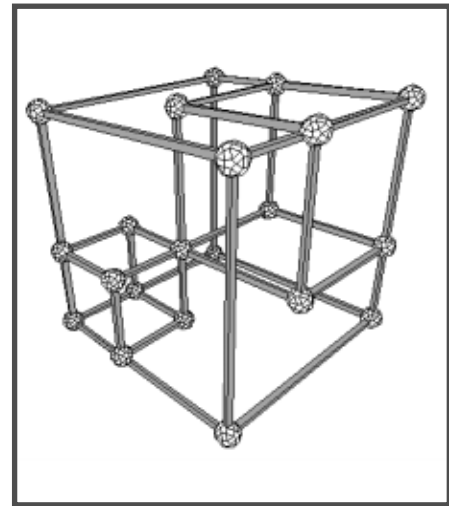
One class period of 45-60 minutes.

Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Getting to Know the World's Greatest Artists: Picasso by Mike Venezia
- Opaque projector if available.

Procedure:

Read the book for the class. As you read elicit responses and comments from students. *What do you notice about Pablo Picasso's style? When was Picasso born? Where was he born? Have you seen any other work that reminds you of his work? Can we always tell what the paintings are supposed to represent?* Highlight the section on *Cubism*. Show the pictures in this section and have students discuss what they notice. (If possible you may want to project them onto a screen using an opaque projector.) Explain that Cubism is an art form, made popular by Picasso, that uses geometric elements such as cubes and angles to give the appearance of the picture being broken into many pieces.



Discuss the difference between 2 dimensions and 3 dimensions. Encourage students to use classroom examples as they create a definition.

Divide the class into teams of 3-4 students, and distribute the Zome System pieces. The first challenge for the teams is to create as many different cubes and distorted cubes as possible in 15 minutes.

When they are finished, students should walk around the room to view the different types of cubes. *How many different versions are possible? Are they really all cubes? Did the color of strut used affect the type of cube produced? How?*

Students should then return to their own cubes. Ask them to close their eyes and touch the cubes. Have them trace around the cube with their fingers. *What different parts do they feel?* As a class define the elements of a cube, including vertex, face, and edge.

Students should individually create a Zome System structure using Cubism principles. When the structure is completed they can draw the structure in their journals. The drawing should identify the vertices, faces, and edges.

Assessment:

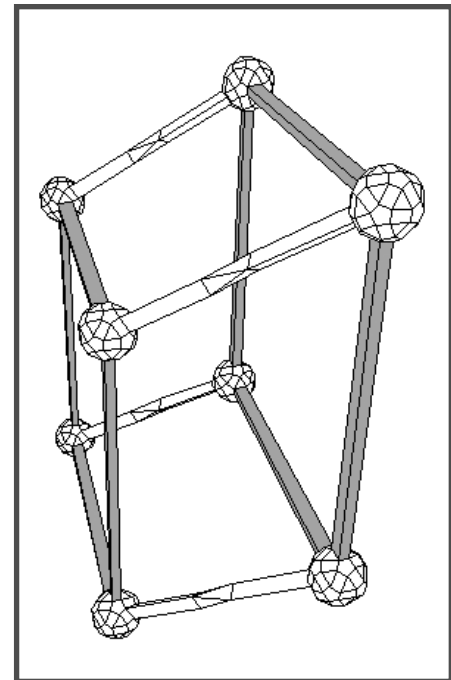
Observe students work, and review their notes and drawings. Students have met the standard when they create various distorted cubes and define their vertices, faces, and edges. To exceed the standard they must construct a Zome System structure using Cubism principles.

Standards Addressed:

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Fine Arts standards addressing the **impact of philosophical and artistic ideas throughout history**.

Transfer Possibilities:

Identification of elements in other geometrical structures ("2-D and 3-D Shapes," "Attention Angles!" "Cubes - I," and "Cubes - II"). Continued work on Cubism using other media. History of other 20th century art movements ("Mondrian Lines").



Zome System

Builds Genius!

Mathematics / Physics / Art Basic Concept

Lesson Objective:

Students will learn about polygons, **parallel lines**, and **prisms**. They will deepen their understanding of the relationship between 2 and 3 dimensions through the use of “speed lines.”

Prerequisite Skills:

Knowledge of basic 2 and 3 dimensional geometric forms (“Geometric Shapes,” “Geometry is All Around Us,” “Shape and Number,” and “2-D and 3-D Shapes”).

Time Needed:

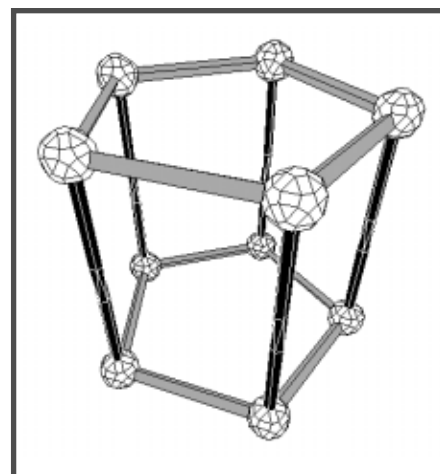
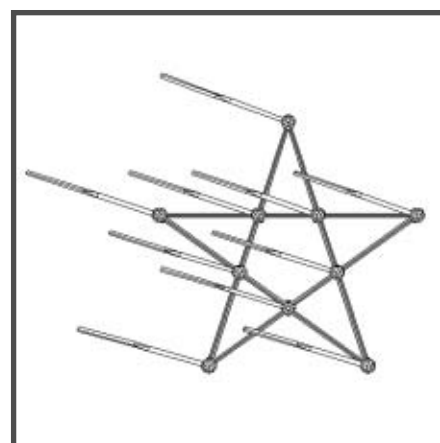
One class period of 45-60 minutes.

Materials Needed:

- Two Zome System Creator Kits for up to 25 students
- Overhead projector

Procedure:

At least one day before this exploration, show your class some illustrations of “speed lines” as they appear in illustration, company logos, etc. Ask each student to bring in at least one example of speed lines from the comics, old yellow pages, packaging, old magazines, the Internet, etc. On the day of the exploration, divide the class into teams of 2 students. Distribute the Zome System pieces evenly. Each team is to build a flat closed shape. A brief discussion on the meaning of “flat” and “closed” may be appropriate as an introduction to the concept of a polygon. For this exploration, it’s enough to know that “flat” means that when the shape is laid on a table, all nodes in the shape will touch the table without being forced, while “closed” means that the nodes and struts form a loop without crossing each other. However, the shapes need not be convex; for example, both the shape of a pennant and a



star are acceptable.

Now ask students to imagine that they are working with Zome System in the space shuttle. They can let go of their shape and watch it float in a straight line from one side of the cabin to the other. Have each student choose a direction for the shape to “float”, and insert a strut in any node to indicate the direction the shape is moving. The strut should not be in the same plane as the shape, i.e. it should not lie flat on the table with all of the other parts in their shape. Ask for one or more volunteers to show how their shapes float through space in the direction of the strut they inserted.

Now discuss the examples of speed lines the students brought in. *Do all speed lines have things in common? Are they all the same length? Do they all go in the same direction? What are speed lines trying to show? Is there a relationship between speed lines and time? Is there a relationship between speed lines and space? Where was the object in the example before it moved? Where is it now? Are speed lines always the same size as the shape they come off of?*

When the class reaches general agreement on the nature of speed lines, challenge them to build speed lines on their shapes to show which direction the shape is floating through space. Next they should put nodes on the ends of all their speed lines and connect the nodes together with struts.

Conclude the exploration with a discussion of the different concepts your students encountered, including speed, parallel lines and prisms (the overhead projector can be useful to discuss particular models). *What happens if a shape floats through space in one direction for a second? a minute? a year? What happens if it's going slowly? quickly? What is the relationship between the shape “on top” and the shape “on the bottom” of your model (if the “speed line” struts are parallel and of the same length, the shapes will be identical)? Does the shape on top have to be the same as the shape on the bottom? Why, or why not? What is the name of an object consisting of two identical polygons connected by parallel lines (a prism)?*

Students should draw pictures of their shapes, with speed lines, in their math journals, and to write down observations and conclusions.

Assessment:

Observe students while they work, and review notes in journals. To meet the standard students must build a prism consisting of two identical polygons with connecting parallel lines. To exceed the standard they must make observations of how speed lines represent movement, velocity, and time.

Standards Addressed:

- * Physical Science standards addressing concepts of **velocity and Newtonian motion**.
- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

More exploration of the relationship between 2 and 3 dimensions in mathematics, art, and science (“3-D Triangles,” “Cubes - I,” “Cubes - II,” “Cubes - IIIA,” “Cubes - IIIB,” “3-D Triangle Tiles”).

Mathematics / Art Basic Concept

Lesson Objective:

Students will learn how to build distorted versions of regular polygons by working with shadows of Zome System models.

Prerequisite skills:

Ability to recognize various polygons and connect them to numbers and symmetries (“Shape and Number,” “What is Reflection Symmetry?” “2-D and 3-D Shapes,”).

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

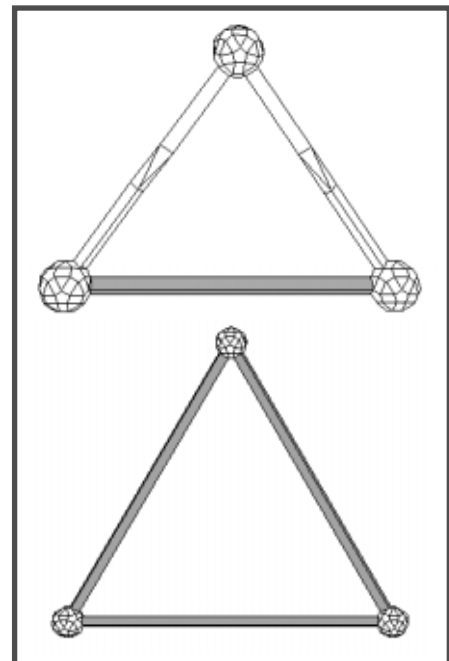
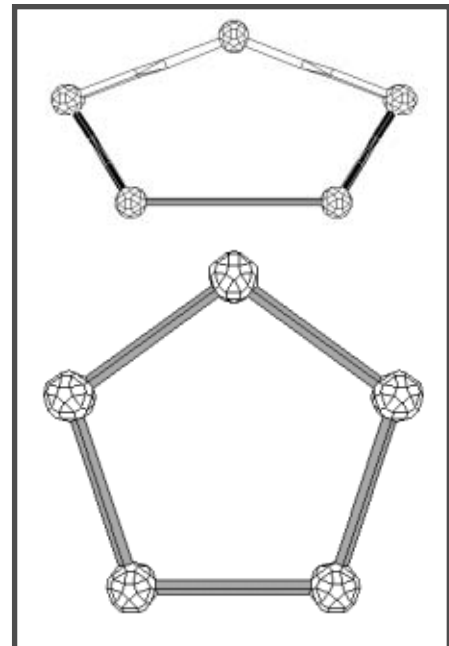
- One or two Zome System Creator Kits for class of 25-30 students
- One or more overhead projectors
- Large sheets of paper in case shadows do not fit in student’s math journals (optional)

Procedure:

Prepare for the class by setting up overhead projectors so many students are able to cast shadows simultaneously. If the class can be held on a sunny day, it is best to hold part of the lesson outdoors; sunlight produces the crispest shadows.

Divide the class into teams of 2 students, and distribute the Zome System pieces evenly. The challenge for the teams is to build squashed or distorted versions of polygons they have worked with in previous lessons. The distorted shapes must be identical to shadows of the polygons they already know.

The teams should first build the regular shapes they learned in the “Shape and Numbers” exploration. One team member should then use the overhead projector to

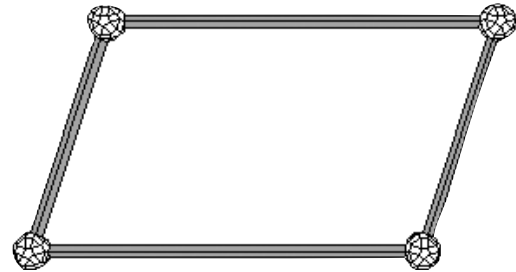


Squashing Shapes

Zome System

Builds Genius!

cast a squashed shadow of the polygon model, while her/his partner traces the shape of the shadow on paper. The partners then switch roles. Both team members should build Zome System models which matches the shadow drawings as closely as possible. When both team members are satisfied with their new shapes, they should return to the light source. One should hold a squashed shape flat against the screen or wall, while the other manipulates her/his regular model, trying to exactly line up its shadow with the squashed model. Switch roles so both members get to understand the necessary manipulation of the model in the light.



When all teams have finished work, ask each team to select one successful shadow match to demonstrate in front of the class. Lead a class discussion of student discoveries. *What makes the shadows squashed? Under what conditions are the shadows not squashed? How can you be sure? Was everyone able to match regular and squashed models exactly? Why, or why not? What is the “most squashed” shadow you can get? Why? What is the same about the regular models and their squashes (number of sides, combined angles)? What is different about the regular models and their squashes? How many different squashes could you build of the same regular model? Is it possible to cast stretched shadows? Is it possible to build stretched models? Is it a stretched model just a squashed model turned sideways?* The class can be extended to include the study of distortions in patterns and pictures in other media. Examples can include pictures of railroad crossing warnings painted on a highways, or very stretched writing from puzzle books. *What do we actually see when we view a TV screen from an angle instead of straight on.*

Assessment:

Circulate and question the teams while they are working. Review their drawings, models, and notes in journals. To meet the standard students must complete at least two squashed models of regular polygons. To exceed the standard they must verbalize how the squashed and regular shapes relate to each other (what is the same, and what is different). They must also develop a theory why the movement of the model in the light produces a distorted shadow.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Fine Arts standards that **identify and apply the elements of art in a variety of media.**

Transfer Possibilities:

This lesson is appropriate as a starting point for work on shadows and perspective drawing (“3-D Triangles,” “Speed Lines!,” “Cubes - I,” “Cubes - II,” “Cubes - III,” and “Cubes - IV”).

Mathematics Basic Concept

Lesson Objective:

Students will learn basic properties of the **cube**, including **parallel** and **perpendicular lines**, number patterns, and **symmetry groups** associated with the cube.

Prerequisite skills:

Familiarity relating geometric shapes and symmetry lines to representative numbers (“Shape and Number,” “3-D Triangles” and “Speed Lines!”).

Time Needed:

One class period of 45-60 minutes.

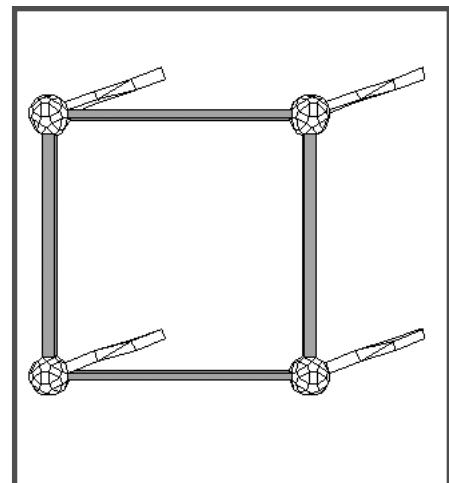
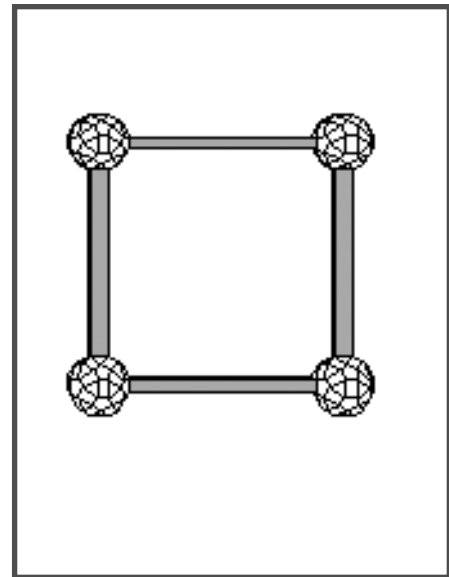
Materials Needed:

- One or two Zome System Creator Kits for class of up to 24 students
- Overhead projector

Procedure:

Divide your class into teams of 3-4 students and distribute Zome System components evenly among the teams. Based on their experiences in the “Shape and Number,” “3-D Triangles” and “Speed Lines!” explorations, challenge each team to build a “3-D number 4.” Students may ask what you mean by a “3-D number 4.” Rather than leading a group discussion at this point, suggest that the teams discuss the concept as they build.

Allow 10-15 minutes for this exploration, during which time you can offer guidance on an individual or team basis. *What was the shape of a 2-dimensional number 2? A 2-D number 3? A 2-D number 5? Why did you decide a rectangle is a 2-D number 2? A triangle is a 2-D number 3? A pentagon is a 2-D number 5? What color struts do you think should be used to build a 2-D number 4? A 3-D number 4? Why?*



Cubes - I

Zome System

Builds Genius!

What was the shape a 3-D number 3 (i.e., a 3-D triangle?) What shapes did you use to build the 3-D triangle? If you know the shape of a 2-D number 4, do you know which shape(s) build a 3-D number 4?

If the students are having problems they can use a step-by-step procedure and start by building a “2-dimensional number 4” based on their answer to the previous questions. *How can you be sure that the square is a 2-D number 4?* To “break into” the 3rd dimension, they can then add “speed lines” to the 2-D number 4. *Which direction should the speed lines go? What color struts should be used?* To complete the jump to “3-space,” put nodes on the ends of the speed lines and connect the new nodes with struts.

At the end of the exploration, ask for a representative of each team to present their team’s “3-D number 4s” to the class and explain why it answers the challenge. Team representatives can use the overhead projector to show a 2-D shadow of their 3-D models.

After the presentation, discuss the various concepts comprising the idea of a 3-D number 4. *How do you know it is a 3-D number 4? How many nodes does it take to make a 2-D number 4? How many struts? How many 2-D number 4s does it take to make a 3-D number 4? How many nodes? How many struts?*

Students should record their observations in their math journals. They can include a table of the number relationships they discover, as shown below.

What is another name for a 3-D number 4? (Cube, hexahedron) If we were to describe this 3-D shape, how would we? (A regular hexahedron has six faces which are all regular polygons. A regular polygon is a convex 2-D shape which is made up of same length lines all meeting at the same angle. For example, a 2-D number 4 is commonly called a square. It consists of a loop of 4 lines of equal length, joined at 4 points at equal (right) angles. A regular polyhedron is a 3-dimensional shape formed from regular polygons which also has equal angles between faces.)

Students should save their cube models for the “Cubes - II” exploration.

As an extension the class can discuss what a 4-dimensional cube would look like? *Is there any way to predict what factors determine the structure of a 4D cube based on the number chart below?*

	Number of points	Number of lines	Number of 2-D squares	Number of 3-D squares
0-D # 4				
1-D # 4				
2-D # 4				
3-D # 4				
4-D # 4				

Zome System

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Cubes - I

Assessment:

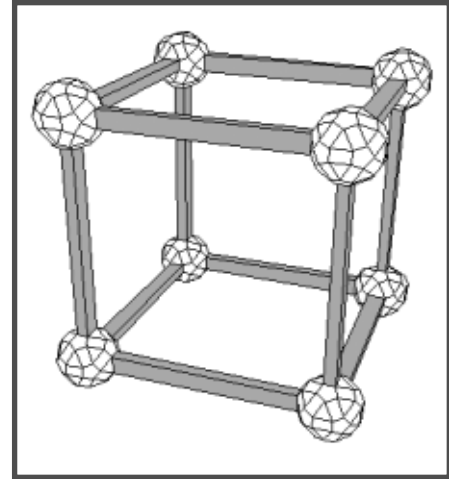
Take notes during the discussions and review written work in math journals. To meet the standard, students should understand the basic properties of the cube, including parallel and perpendicular lines, number patterns, and symmetry groups associated with the cube. To exceed the standard, students will be able to extrapolate to four-dimensional figures.

Standards Addressed:

- * Mathematics standards addressing number sense and numeration (NCTM Standard 6).
- * Mathematics standards addressing geometry and spatial sense (NCTM Standard 9).

Transfer Possibilities:

Continued work on perspectives of the cube and perspective drawing ("Cubes - II, III, and IV" lesson plans.)



Mathematics Basic Concept

Lesson Objective:

Students will learn about the numbers 2, 3, and 5 in relation to the cube by using shadows.

Prerequisite skills:

Ability to recognize various polygons and connect them to numbers and symmetries in 2 and 3 dimensions (“Shape and Number,” “What is Reflection Symmetry?” “2-D and 3-D Shapes,” “3-D Triangles,” and “Speed Lines!”). Ability to build and describe the properties of the cube (“Cubes - I”).

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

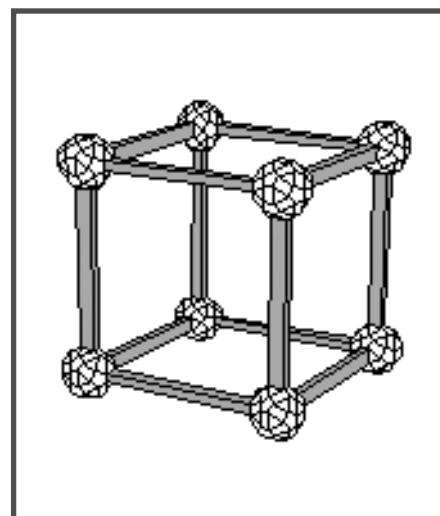
- Two Zome System Creator Kits for class of 25-30 students
- Overhead projector or sunlight
- Shadow casting surface (screen for overhead projector; white cardboard for sunlight)

Procedure:

Regroup the class into the teams from the “Cubes - I” exploration. Each team member should have a cube model (one small, one medium, and one large cube per team) built during that lesson.

The challenge for the teams is to discover and draw as many different kinds of “special shadows” of their cube models as they can. Explain that you can cast special shadows by putting a long strut into a node in the model and holding it so that the long strut casts the smallest possible shadow (it will be completely blocked by the shadow of the node.)

One way to help students understand this concept is to discuss “no-shadow days” in the tropics. On certain days



Cubes - II

Zome System

Builds Genius!

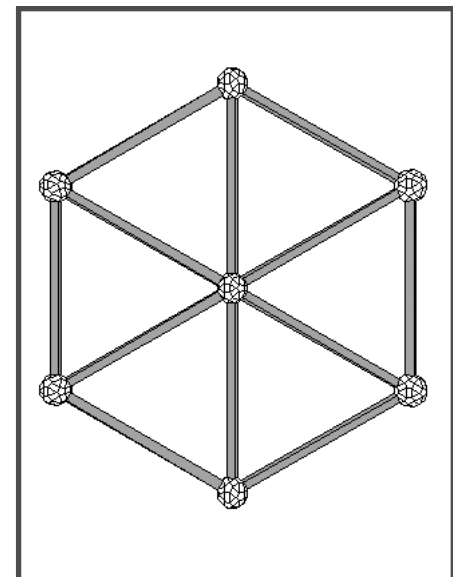
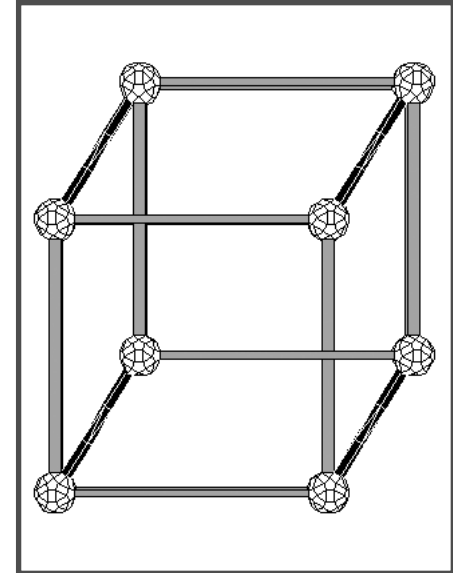
of the year you won't cast a shadow if you're standing in the noon-day sun. This can be demonstrated with a globe, a Zome System strut representing someone standing in the tropics and a plane light source like an overhead projector or sunshine. *Under what conditions do "no shadow days" occur on planet earth? Why does it happen around noon? Is it affected by daylight savings time? Why do you have to be standing up? What is the relationship between the sun's rays and the surface of the earth? What is the relationship between the sun's rays and the person standing outside on a no shadow day? Why do they happen in the tropics? How many times a year do they happen in one place?* Students will learn through experience that the light rays need to be perpendicular to the screen/cardboard and parallel to the strut casting no shadow in order to be effective.

The teams may divide the work, with one team member in charge of putting different color struts in different holes in the cube models, and tracking which combinations have been tried, another casting the shadows, and a third sketching each new shadow in her math journal.

The sun is a very good plane source of light, but it's more difficult to be sure the light rays are perpendicular to the shadow casting surface. Challenge the students to find a way they can be sure the shadow casting surface is perpendicular to the sun's rays. *What if you put one of the shapes you made in the "Shape and Number" class (with the middle strut sticking straight up) flat on the shadow casting surface? When is shadow casting surface perpendicular to the sun's rays? Once the shadow casting surface is perpendicular to the sun's rays, how can it be kept in place?*

Allow 25-30 minutes for this exploration. Circulate and encourage students to find as many different special shadows as they can, making sure they all meet the requirement that a strut inserted in one of the nodes in the cube model casts no shadow. Also encourage teams to classify their findings in a way which will make presentation to the class easier.

When the exploration is over, ask one representative from each team to present their findings. *How many different kinds of shadows could you make with a blue strut? a yel-*



Zome System

Builds Genius!

Cubes - II

low strut? a red one? Which were the favorite shadows? Why? Is there any relationship between the shape of the strut and the resulting shadow? Some of the students will notice that a Zome System projection along a 2-fold axis of symmetry (blue struts) will often exhibit 2-fold symmetry. Likewise, projections along 3- and 5-fold axes of symmetry (yellow and red struts) will often exhibit 3- and 5-fold symmetries, respectively. While students may not need or even want the vocabulary to describe this discovery, they will certainly gain an intuitive understanding.

Assessment:

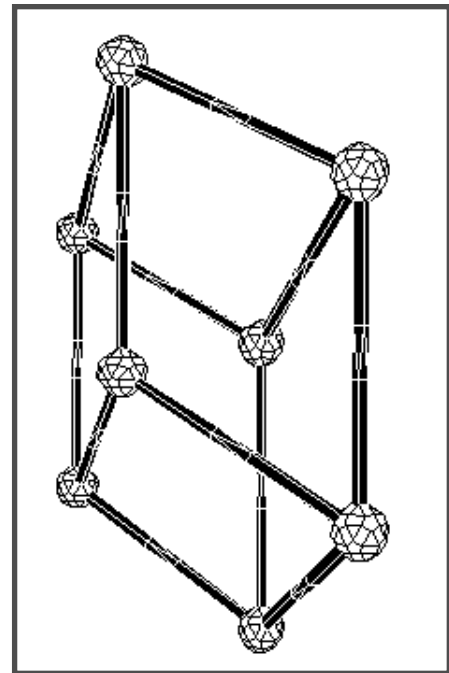
Take notes during discussions and presentations, and review drawings of shadows. To meet the standard, students must create and describe a series of shadows of the cube. To exceed the standard, they must correctly identify certain shadows as representing 2-fold and 3-fold symmetries.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **number sense and numeration** (NCTM Standard 6).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).
- * Physical Science standards requiring students to **know and understand common properties, forms, and changes in matter and energy.**

Transfer Possibilities:

Continued exploration of symmetry concepts (“Multiple Reflection Symmetry,” and “Rotational Symmetry”). Also useful as a starting point for more work on shadows and perspective drawing.



Zome System

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Cubes - III

Mathematics / Physics / Art Basic Concept

Lesson Objective:

Students will deepen their understanding of the relationship between 2, and 3 dimensional objects by working with the shadows of the cube.

Prerequisite skills:

Ability to define the difference between 2-dimensional and 3-dimensional objects ("2-D and 3-D Shapes," "3-D Triangles," and "Speed Lines!"). Ability to build and describe the properties of the cube ("Cubes - I").

Time Needed:

One class period of 45-60 minutes.

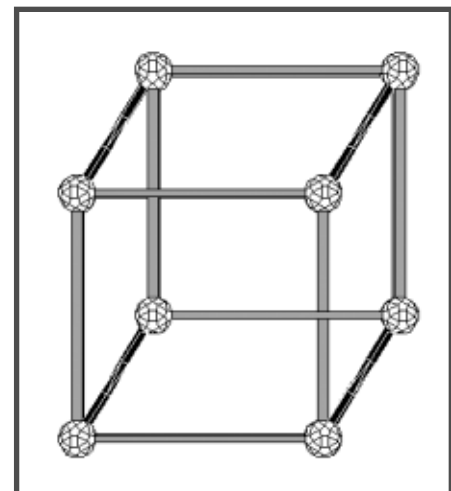
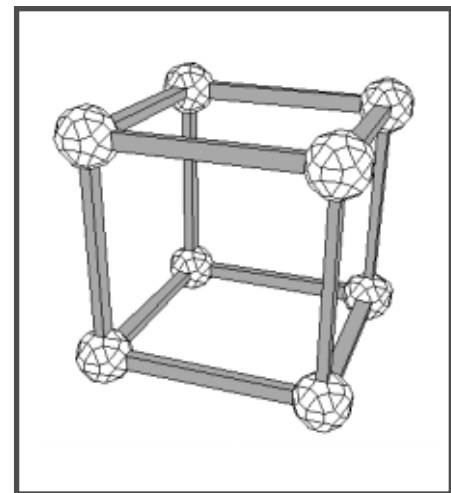
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- One 9"x12" sheet of blueprint paper per team of 3 students (see "Resources" section)
- One piece of 12"x 15" cardboard per team, to cast shadows on
- One cardboard box per team, large enough to fit over Zome System model and sheet of blueprint paper
- Household ammonia solution
- One 9x12 plastic tub or cake pan (not aluminum)
- Masking tape

Procedure:

Prepare the supplies needed by following the instructions in the "Resources" document. The required ammonia solution can be harmful, the "Caution" and "First Aid" notices should be taken very seriously. Cut the large sheets of blueprint paper into the required 9"x12" sheets.

Since students will be working with a photosensitive material, the work area should be set up in an area out of direct sunlight. The paper will be exposed by direct sun-



light in a few seconds, fluorescent light in less than an hour, incandescent light will take several hours.

Regroup the class into the teams from the “Cubes - I” and “Cubes - II” explorations. Each team member should have a cube model (one small, one medium and one large cube per team) built during those lessons.

The challenge for the teams is create a “permanent shadow” of one of their cubes, using sunlight and blueprint paper. They must take notes of all their findings as they work. First they are to attach their sheet of blueprint paper to a piece of cardboard using masking tape. A cube model should be placed on the blueprint paper (can be anchored with pins if desired). *Where on the paper should the cube be placed in order to center the shadow on the paper? Which size cube should be used? Where in the sky will the sun be?*

Place cardboard boxes over the cube and blueprint paper, and carry the whole assembly into direct sunlight. Remove the boxes and let the paper fade from yellow into white (apx. 3-5 seconds). Re-place the box over the model and paper, and remove from sunlight. “Develop” the images following instructions in the “Resources” document.

When all teams have successfully produced cube shadows they can be posted in the class room for viewing and discussion. *Are all the shadows the same? Why, or why not? Do all the shadow have something in common? What do they have in common with the cube model? How can all the cubes be the same, and their shadows be different? How many squares are in every shadow? How many diamonds are in the shadows? Do squares count as diamonds too? How many lines are in each shadow? How many points? Is it possible to build a Zome System model other than the cube that would line up with the shadow created today?* The answer to the last question will be found in the “Cubes - IV” lesson.

Assessment:

Take notes during discussions and while the students experiment, Review the created “shadows” and notes taken by the teams. To meet the standard, students must create and analyze permanent 2-D images of a cube.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4)
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9)
- * Physical Science standards requiring students to **know and understand common properties, forms, and changes in matter and energy**
- * Fine Art standards addressing **perspective and projection drawing**

Transfer Possibilities:

More work on shadows and perspective drawing.

Zome System

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Cubes - IV

Mathematics / Physics / Art Basic Concept

Lesson Objective:

Students will deepen their understanding of the relationship between 3 dimensions and 2 dimensions by working with shadows of the cube and other models.

Prerequisite skills:

Ability to define the difference between 2-D and 3-D objects ("2-D and 3-D Shapes," "3-D Triangles," and "Speed Lines!"). Ability to build, describe the properties of, and create permanent shadows of, the cube ("Cubes - I," "Cubes - II," and "Cubes - III").

Time Needed:

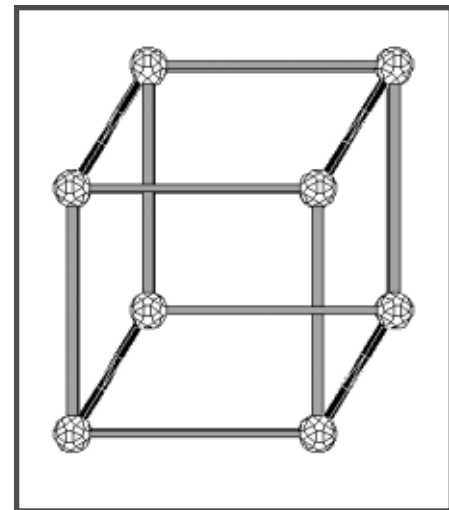
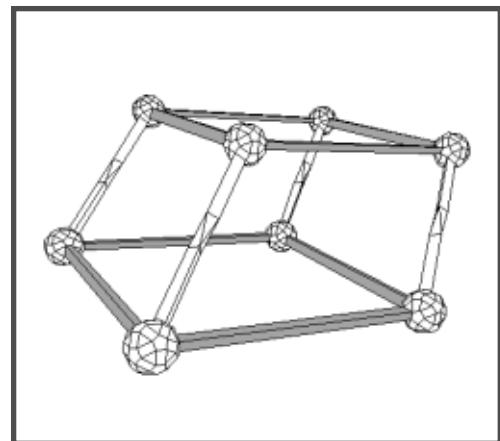
One class period of 45-60 minutes

Materials Needed:

- Two Zome System Creator Kits for class of 25-30 students
- Blueprint cube "shadow" for each team, as created in the "Cubes 3" exploration
- One piece of 12"x15" cardboard for each team of 3 students, to cast shadows on
- Overhead projector(s), or lots of sun
- Books, posters, slides or other materials on perspective drawing and optical illusions. Artwork by M.C. Escher is recommended.

Preparation:

Organize the classroom so several teams of students project shadows simultaneously. Set up a single overhead projector on the far side of the room to light up an entire wall, or use several overhead projectors. Alternatively students can work in a sunny area of the classroom or outside. Regroup the class into the same teams of 3 from the



“Cubes 3” lesson. The challenge for the teams is to build a Zome System model other than a cube which casts the same shadow as they captured on blueprint paper in the previous class. Each team member should build one model which s/he thinks will answer the challenge. Team member should rotate through the roles as Scribe, Holder, and Shadow.

The Scribe directs the trial of her own model and records the results in his math journal, The Holder holds the Scribes' model in various positions between the light source and the shadow casting surface, and the Shadow holds the shadow casting surface (with the blueprint cube shadow taped to it) in various positions. The Holder and the Shadow cooperate to cast the shadow of the 3-dimensional model directly on top of the blueprint cube shadow. The Scribe directs by suggesting changes in the position of the model or the shadow casting surface until the 2 shadows match up. If no match can be found after five minutes, teams should rotate roles and try the next Scribe's model. When all teams have finished their three rotations, each team selects one successful shadow match to present to the class.

Discuss of student discoveries during and after the presentations. *Were all the cube shadows from the previous exploration the same? Why, or why not? Is it possible to make a model other than a cube which casts the same shadow as a cube? More than one? How many? Do the new models have in anything in common with the cube models? How many struts does each successful new model have? How many nodes? Could you make a flat model which casts the same shadow as your blueprint cube shadow? Is it easier or harder? Why? What happens to shadows when you rotate the shadow casting surface away from the light? Why? What happens to shadows when the sun moves through the sky? Could two people look at the same drawing and see two different things? Why?*

Continue the exploration of perception, perspective, dimensions, and optical illusion, using artwork by M.C. Escher and others. Drawing and perception are all related to the jump from 3 dimensions (an object) to 2 dimensions (an image) and back to 3(+) dimensions (the object as it exists in the mind). Students can discuss both practical application of these concepts, and more abstract philosophical aspects.

Assessment:

Take notes during discussions and while the students experiment, Review the created “shadow models” and notes taken by the teams. To meet the standard, students must build Zome System models which cast shadows identical to those they created earlier using cubes.

Standards Addressed:

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4)
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9)
- * Fine Art standards addressing **perspective and projection drawing**

Transfer Possibilities:

More work on shadows and perspective drawing.

Zome System

Builds Genius!

Even and Odd Numbers

Mathematics Basic Concept

Lesson Objective:

Students will learn the concept of even and odd numbers in relation to the rotational symmetry of 2-dimensional polygons. The concept of **geometric projection** will be introduced.

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

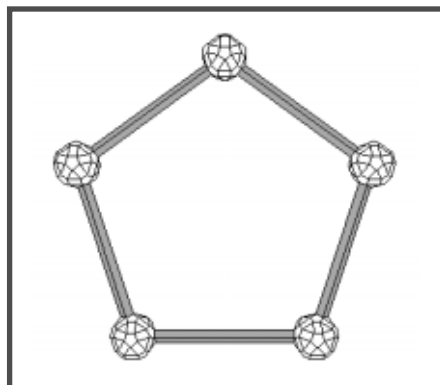
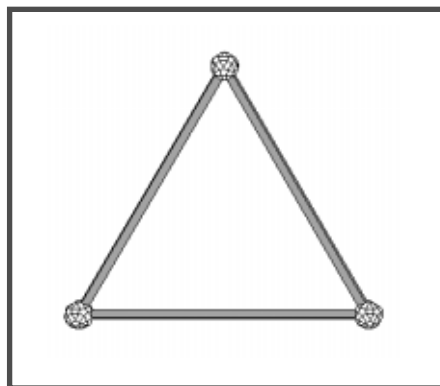
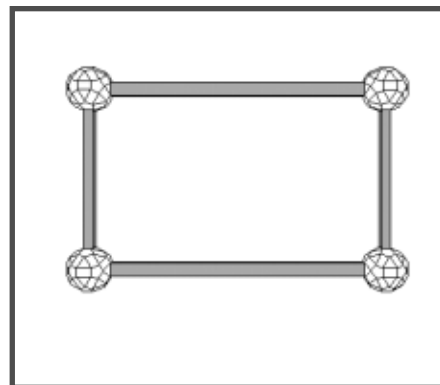
- One or two Zome System Creator Kits for 30 students
- Overhead projector or blackboard

Procedure:

Divide your class into teams of 3 students each, and distribute the Zome System components evenly among the teams. Initiate a short discussion about the components. *Which shapes are the holes in the node? Are all the struts the same lengths? How do the 3 types of struts differ from each other? Students will point out that the red and yellow struts are “twisted” while blue struts are “straight”. Why do you think this is so?*

Ask the teams to build the cross-sectional shapes of the struts based on their experience in the “Shape and Number” exploration. Team members should build the three shapes, Golden Rectangle, equilateral triangle, and regular pentagon. Encourage students to use shorter struts as they later will trace them in their math journals.

The next challenge is to find the center of those shapes using Zome System parts. The question may arise whether the central node must lie flat on the table with the other nodes. This is possible with the rectangle but not with the triangle or pentagon. When a student builds a possible solution, the teacher can place it on the overhead projector and ask if this solves the problem of find-



Even and Odd Numbers

Zome System

Builds Genius!

ing the center of the shape. *“How does the shadow ‘project’ the node to the center of the shape?”* Ask your students to lay their shapes flat on the table. Can they find a strut which they can insert in the central node so that it sticks straight up from the shape? As shown in “Shapes and Numbers,” the cross-sectional shape of the central strut is the same as the shape on the table.

How would we go about finding the central point on a blank sheet of paper? The rectangle model and use of a ruler should help students find a simple solution. Now ask them to choose one of the 2 lines on the paper and call it the “center line,” and place the model over the center point so that one of the “spokes” lines up with the center line. The teams should mark one of the struts with a piece of tape. Also ask them to trace the model onto the page. *How many sides of the rectangle are on one side of the center line?* They should record the number in their journals.

Challenge the teams to find a way to turn the shape half-way around while keeping the center node over the point in the center of the page, and then trace the result. Use a clock analogy to illustrate: at the top of the hour, the big hand points to 12; half-way around it points to 6. Students can write 12 at one end of the center line and 6 at the other and call the strut with the tape the “big hand.” The teams should then repeat this process for the other two shapes on separate pages in their journals, and write down their observations.

Allow about 10 minutes at the end of the period to discuss their observations. Your students will have discovered that when turned half way around, the rectangle (number 2) lines up with itself while the triangle (number 3) and the pentagon (number 5) make “stars.” The rectangle has 2 struts on one side of the center line while the triangle has 1.5 and the pentagon has 2.5. Students should conclude that odd numbered struts are twisted while the even numbered strut is straight.

An excellent extension challenge is to build a copy of the Zome System node. This larger model makes it easy to see how the triangular and pentagonal holes on opposite sides of the node are rotated relative to each other. The odd number struts need the twist so the nodes in a structure remain in the same orientation.

Assessment:

Review written work and drawings in math journals. Question students individually and in groups to ensure that they understand how the different shapes react to the 180° rotation. To meet the standard, students must complete the drawings of the rotated polygons. To exceed the standard they must conclude that odd numbered struts are twisted while the even numbered strut is straight.

Standards Addressed:

- * Mathematics standards addressing **number sense and numeration** (NCTM Standard 6).
- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9).

Transfer Possibilities:

Some students may discover that if you switch the central strut to the other side of the shape, you get a cool “spinner.” This is a good introduction to the Spinners lesson plan, and may be used as a follow-up.

Zome System

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Descriptive Writing

Language Arts / Mathematics Intermediate Concept

Lesson Objective:

Students will use geometric models to practice technical writing.

Prerequisite Skills:

Students need to have played with Zome System before.

Time Needed:

One class period of 45 to 60 minutes.

Materials Needed:

- One or two Zome System Creator Kits for class of 25-30 students

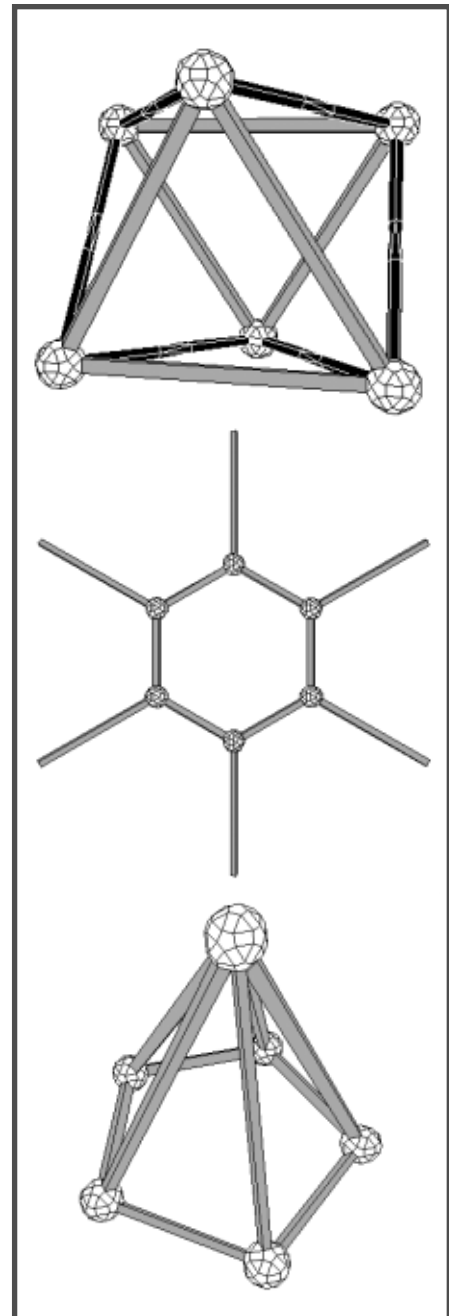
Procedure:

Introduce the lesson to the class as a practice session in constructing models from written descriptions.

Divide the class into pairs, and distribute the Zome System pieces. The pairs should re-arrange their seats so they are sitting back to back. Their task is to build small models based on descriptions written by their partner. Each student should first spend 10 minutes building a simple Zome System model, and writing a detailed description of it. The partners then exchange descriptions, taking care that they do not show the models. They then spend 5 minutes trying to re-create their partner's model. When the time is up, the pairs should compare the original models to those built using the descriptions. *Are the models identical? If not, why? Are the models built according to the descriptions? What was missing in the description to make a perfect copy?*

Following their discussion the student pairs should repeat the entire process and see if their discussion helped them write better descriptions

Bring the entire class together to share their experiences,



and discuss implications of the exercise. *Was this a difficult exercise? Why or why not? Were some types of models easier to describe than others? Which element needed to be present in the descriptions? Was it necessary to know any specific geometry vocabulary to write a good description? Would drawings have helped? What are practical applications of this type of writing skills? Which professions are required to be good at descriptive writing?*

The final task for the student pairs is to write a short manual of how model descriptions should be organized.

Assessment:

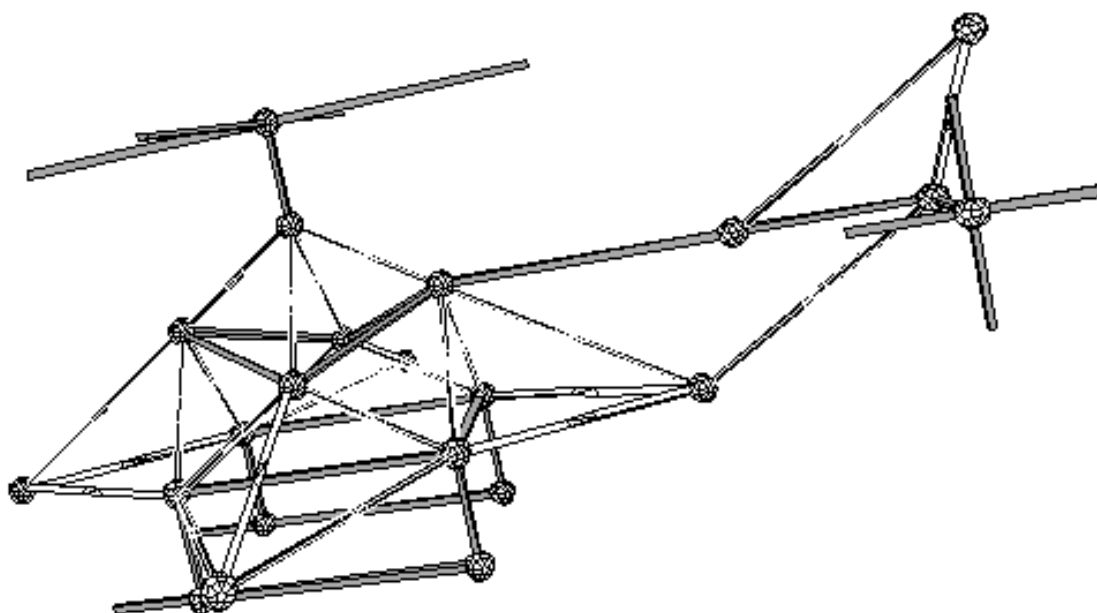
Take notes while the students build and write their descriptions. Review their “description manuals”. To meet the standard, students must write a description clear and concise enough to allow their partner to build a copy of the Zome System model. To exceed the standard, they must produce a “manual” with a standardized method of writing model descriptions. They must also draw connections to real-life applications of descriptive writing skills of this type.

Standards Addressed:

- * Language Arts standards requiring students to write and speak for a variety of purposes
- * Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Working on geometric vocabulary (“Naming 2-D and 3-D Shapes”). Technical writing in connection with construction projects (“Bridge Building Unit”).



Zome System

Builds Genius!

Bubbles II: Minimal Surfaces

Physics Intermediate Concept

Lesson Objective:

Students will understand the geometry of soap bubbles joining together, and that the shapes formed by bubbles are derived from laws of surface tension and minimum surface area relationships.

Prerequisite Skills:

Students should have previous experience making bubbles with Zome System frames (“Beginning Bubbles”). Ability to relate symmetry concepts to specific angles and geometric shapes (“Shape and Number,” “What is Reflection Symmetry?” and “Multiple Reflection Symmetry”). Previous experiments and/or discussion about surface tension.

Time Needed:

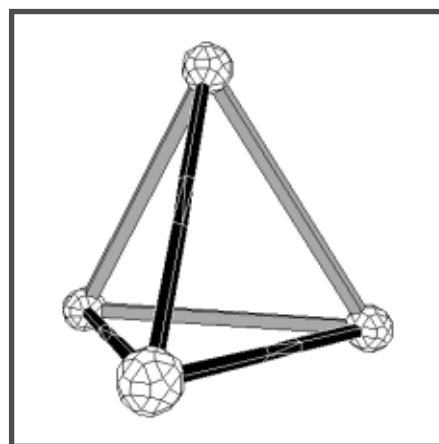
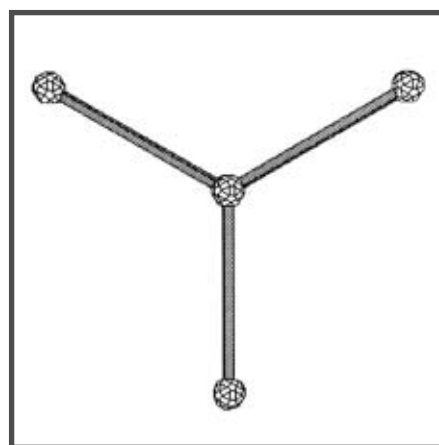
One or two class periods of 45-60 minutes.

Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Three or four buckets of water with apx 1/5 cup of liquid dish soap (Dawn works well)
- A few drinking straws
- One balloon (round)
- Two small pulleys and string

Procedure:

Prepare by mixing the bubble solution if you have not saved the solution from the “Beginning Bubbles” class. Place the buckets in your classroom so the students have space to work around them. Place newspapers on the floor and workspace to absorb spills. Initiate the class with a discussion about how natural systems are affected, if not driven by, least energy considerations. *Can the class think of a good example?* One example is a river. It will always follow the path of least energy to the sea. It can’t go up and over a mountain when it can



Bubbles II: Minimal Surfaces

Zome System

Builds Genius!

go around. This is not laziness, it is simply a consequence of natural law whereby nature expends the least energy possible.

The soap bubble is another good example of this. *What shape are most soap bubbles? Why is it shaped like a sphere?* Allow some discussion on this topic.

A balloon, which works according to the same principles as a bubble, can be used as a representation of the soap bubble. As you blow up the balloon, the surface tension increases, the skin of the balloon becomes tighter, pulling away from itself in every direction on the sphere. As with bubbles, if the skin becomes too thin, the balloon will pop. The bubble is at the equilibrium point between the surface tension of the soap film (pulling inward) and the interior air pressure (pushing outward).

So, why are the bubbles in the shape of a sphere? Why not a cube or a pyramid? The bubble wants to have the least surface area on the outside with the most interior volume for the entrapped air. The shape with the highest volume with the least surface area is the sphere. It is also symmetrical from every possible direction, demonstrating the equal pressure and tension on the bubble.

Where else do we see this balance of internal and external forces?

When bubbles join they are trying to pull equally on each other even if they are different sizes. The teacher can use an easily pre-made apparatus, consisting of two small pulleys and string, to explain. Tie a weight to a weight of some kind. Then tie two other strings to the end of the string holding the weight. Run the two strings over the pulleys.

No matter how we move the pulleys around, the strings continuously adjust their position to maintain 120° angles between them. Soap bubbles do this too; when a change is made, either the addition or popping of a bubble, the whole cluster rearranges until equilibrium is reached. The students can use 3 blue Zome System struts at 120° angles to check the angle of the strings.

Next, have each student draw six sets of dots in a triangle as shown. Challenge the class to find the shortest path connecting the three dots. They can measure the total path and compare them. *Which is the shortest path?* Try the same test with four and five dots. *What do all the shortest paths have in common (three way corners)?*

Complete the class by letting students continue their exploration of bubbles with the aid of Zome System structures. Allow them to build simple 3-dimensional structures and dip them in the bubble solution. Circulate and assist the teams as necessary, while calling attention to discoveries made by the students. *Why do the bubbles not stay on the outside of the frame (the bubble films that join inside the structures are smaller than those on the outside)? How does this relate to the balloon? How do the bubble films join together in the frames (each line in the bubble shape is created by three films meeting at exactly 120°)?* Discuss how the bubble lines in a tetrahedron or a triangular prism represent the shortest distance between the nodes in the structure. *What is a practical application of this kind of knowledge (a good example is a cable company looking for the most economical way to connect a number of cities)? Why do the lines not meet in the center of a cube or a pentagonal prism (there would be too many bubble films meeting in one point)?*

Assessment:

Review notes in student journals, and ask team representatives to present ask the group's findings. Students will have met the standards if they can verbalize how bubbles exemplify minimum energy

Zome System

Builds Genius!

Bubbles II: Minimal Surfaces

use in nature. To exceed the standards they must be able to define how the least energy principle is modified by the natural 3-fold symmetry of the bubble.

Standards Addressed:

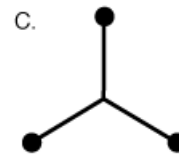
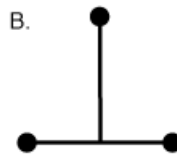
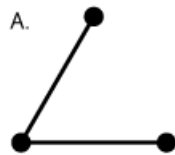
* Science standards addressing **consistency of natural laws**, particularly benchmarks on surface tension and energy use in nature.

Transfer Possibilities:

Physics classes dealing with surface tension, and minimum energy use in nature. Mathematics classes using geometry to address building economics.

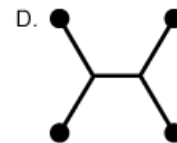
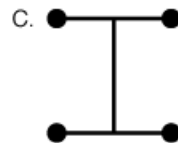
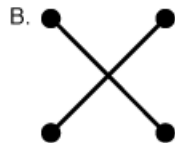
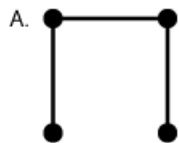
Possible Paths and Shortest Paths

3 Points



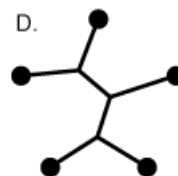
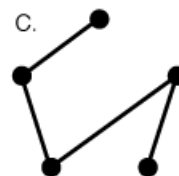
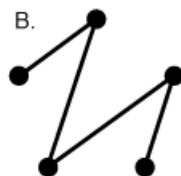
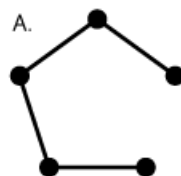
Shortest

4 Points



Shortest

5 Points



Shortest

Zome System

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Tallest Tower in the World

Architecture / Physics Intermediate Concept

Lesson Objective:

Students will start exploring structural stability in architecture. They will understand the importance of **triangular reinforcements**.

Prerequisite Skills:

Ability to identify names and shapes of regular polygons. Some background in the idea of gravity forces.

Time Needed:

Two or three class periods of 45 to 60 minutes

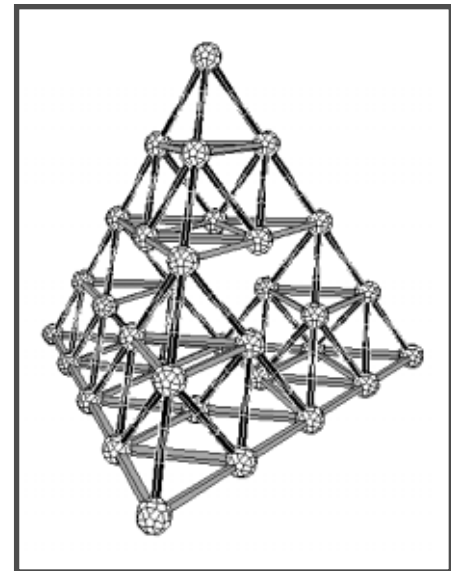
Materials Needed:

- One Zome System Creator Kit per 5-7 students
- Six or 7 telephone books or other heavy books
- Posters or books showing architectural elements such as **trusses, buttresses, and space frames**.

Procedure:

Start the class with a discussion of where we use geometry in our everyday life. Some students will be aware that knowledge of this subject is important to the engineers and architects who design and build houses, bridges, towers, etc. *How do these people know which structures to use in their work? Are some shapes more common than others? Is it important how attractive the shapes are? Are some shapes easier to manufacture than others? Are some shapes stronger than others?*

Divide the class into teams of 4-5 students. Their assignment is to build the tallest tower possible that is able to support a certain amount of weight, such as 3 phone books. Older students can be given additional requirements, such as having to work on a set budget (see the worksheet entitled "Construction Crew Wanted" that is part of the "Bridge Building Project" integrated unit).



Tallest Tower in the World

Zome System

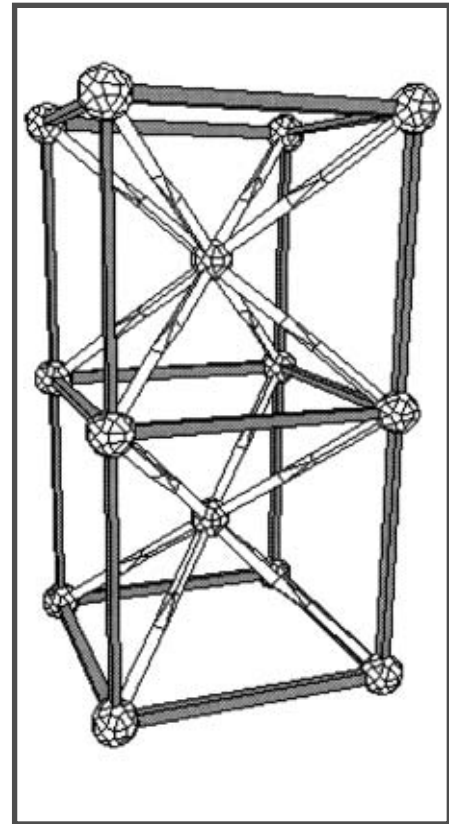
Builds Genius!

Discuss the planning and pre-building design process the teams need to engage in. Set a time limit for their first design.

Circulate and question the students as they work. *How can we test the strength of a shape before it is incorporated into the tower? Is it better to use longer struts as opposed to shorter ones? Where will the tower break, at the connection between the node and the strut, or will one of the elements actually break?* Students should take continuous notes of their observations during the construction phase. After the initial designs have been built each group should present their design to the class. Test the towers one by one, starting with the tallest one, until you have identified those that are able to carry the pre-determined weight. Discuss the appearance of the towers as the phone books are loaded on. *Is it wobbling side to side? Is it starting to twist? Where does it appear to be weak?* Finally, load on additional phone books on the surviving towers to determine the breaking point for each structure.

Lead a discussion of what shapes and reinforcing strategies worked well, and which ones did not. Students should write their own definitions of pattern, **modularity**, and stability. As a class, discuss these elements in the tower designs. *What shapes were most effective in reinforcing the structure? What were some things that the tallest towers had in common?... the sturdiest? ...the most efficient? How would you change the design in a second tower? What aspects would you leave the same?* The teams can then be allowed to build new towers to test their design changes.

The major concept that all students should find is that triangular reinforcements distribute weight most efficiently. *What is it about triangle that makes it stronger than, say, a rectangle or a square? How can we investigate this?* A good way of demonstrating the qualities of these shapes is to build a triangle and a rectangle and apply pressure on them with your hand. The angles of the rectangle will change while the lengths of its sides remain the same. It will thus fold over until it breaks apart. In the triangle the angles and side lengths are fixed to each other, it cannot



Zome System

Builds Genius!

Tallest Tower in the World

fold over. Increasingly large amounts of force can be distributed down along the sides, until the entire shape eventually snaps apart.

When they the teams are finished with their towers, they should write out a chart in three columns, labeled pattern, modularity, and stability, and list the properties of both towers. They should write a definition of triangular reinforcement.

Assessment:

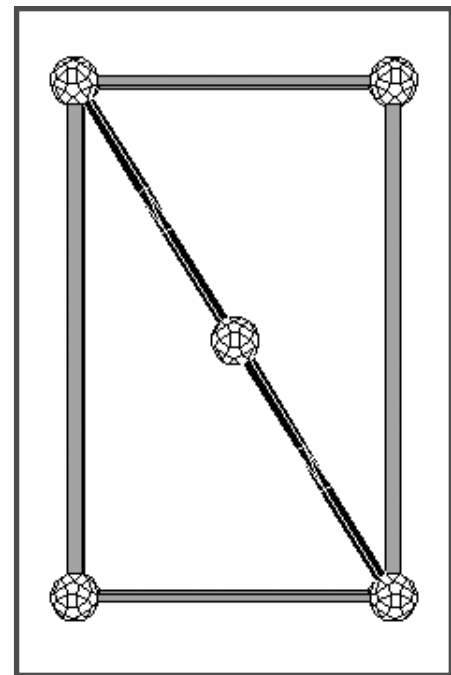
Observe students as they build. Assign a short report where students describe their structures and discuss the effects of different designs. To meet the standard students must identify that triangular reinforcements improves the structural stability of their designs. To exceed the standard they must define in writing why the triangles is stronger.

Standards Addressed:

- * Technology standards addressing structural stability.
- * Physics standards addressing the effects gravity and stress on a structure.
- * Mathematics standards addressing **investigation of mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Further discussion and modeling of triangle patterns, gravity tension, and compression. The activity is also an excellent lead-in to units on architecture and engineering as well as civics discussions on city planning ("Livable City").



Zome System

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The Livable City

Civics / Economics / Architecture Intermediate Concept

Lesson Objective:

To develop a model from which students can explore issues of city organization, social issues, economics, and industry.

Prerequisite skills:

Prior experience building with Zome System.

Time Needed:

3-4 hours for building and 1-2 hours for discussion.

Materials Needed:

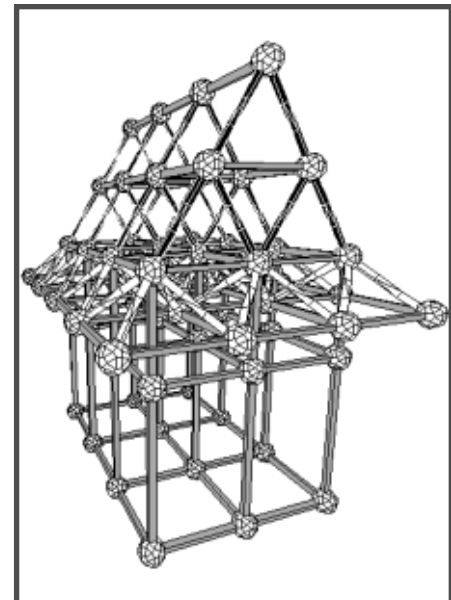
One Zome System Creator Kit per 10-12 students

Procedure:

Begin with a discussion of the town your students live in. *How is our city laid out? Are the streets neatly organized into rows? Where are the stores and industries and homes located? How did our town come to be laid out the way it is?* Explain that this lesson is to explore how cities are organized.

Mark out an area of the floor, about 25 square feet. This is the land area for the city. The goal of this activity is for everyone to have a place to live. A place to live is defined as a space completely enclosed by the Zome System nodes. One way to begin the game is to randomly assign the resources available to the students. Give some students space on the floor but no pieces, some students pieces but no floor. Those with material resources can decide how they are going to be used but cannot build with them. For that they must recruit students who have no materials. A goal of the game can be that each person must have a place to live by the end of the unit. This goal is of course open for discussion.

Was anyone left without a home? Why were they homeless (ask both them and those who were not)? If anyone was homeless, how



The Livable City

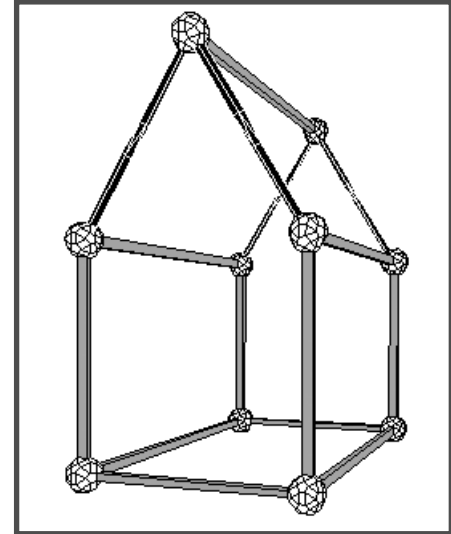
Zome System

Builds Genius!

did that make them feel? Did anyone build themselves a bigger house than they needed to? Why? How did other people react to it? How do you think you might be able to provide for everyone? Do you think that this is possible in the outside world? What was the most efficient means of housing everyone? Individual houses or large complexes? Is it desirable to house everyone? How does their city compare to the one the students live in? Discuss differences and similarities and why those may be as they are. How does this activity relate to economics? Were basic economic principles used in the exchange of land for supplies? What social issues were encountered in the exercise?

Variations:

Older students can have a more complex range of features to deal with, for instance by including the concept of industry. The teacher can also suggest that in order for a space to be livable it must have a power source. Students can build each building with a power source, or build a centralized plant, or several decentralized plants. Everything must be physically represented by some combination of tools. This brings out the ideas of resource use, intelligent design, and economic problems. Students should have the time and space to develop their own world as the game unfolds.



Assessment:

Take notes during both building phase and the group discussion. If possible assign a reflective essay. To meet the standard, each student should end up with a home (or should they?) and a basic understanding of the economic and social issues involved in the activity.

Standards Addressed:

- * Mathematics standards addressing **mathematical problem solving as a method of inquiry and application** (NCTM Standard 1).
- * Social science standards addressing **societal resource distribution, economics, and ethics of government.**
- * Language Arts standards requiring students to **write and speak for a variety of purposes.**
- * Language Arts standards requiring students to **apply thinking skills to their reading, writing, speaking, listening, and viewing.**

Transfer Possibilities:

Continued work on architecture and building economics (“Bridge Building Unit,” and “3-D Triangle Tiles”). Discussions on city planning, resource distribution and taxation, ethics and philosophy of societal organization.

Mathematics Intermediate Concept

Lesson Objective:

Students will learn about three prime factors through deductive reasoning.

Prerequisite Skills:

Familiarity with concept of prime numbers. Students should also have worked on relating geometric shapes to numbers (“Shape and Number”).

Time Needed:

One class period of 45-60 minutes.

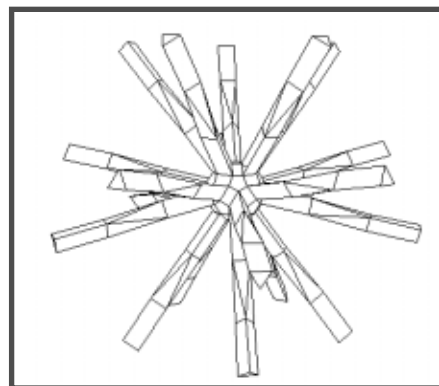
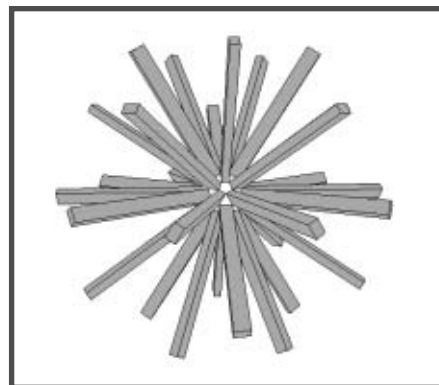
Materials Needed:

- One or two Zome System Creator Kits for class of 25-30 students.
- Standard (analog) clock

Procedure:

Divide your class into teams of 4 students. Give each team 4 Zome System nodes and struts in one length in all 3 colors. For example, one group will get 4 nodes and short red, yellow, and blue struts. It is best to use short and medium struts, although the long will also work, if the class is very large.

Hold a brief discussion of the Zome System node. *How many different shapes of holes are there in the nodes? What are the shapes? How many holes are in each node? What would be an effective way of counting the number of holes?* Let the teams start this problem solving exercise. Several of the teams will divide the work into counting rectangular holes, triangular holes and pentagonal holes. They will find all of one particular kind of hole by building a “pincushion;” that is, putting one color strut in every corresponding hole in the node. Encourage struggling teams to adapt the pincushion method.



Prime Factors

Zome System

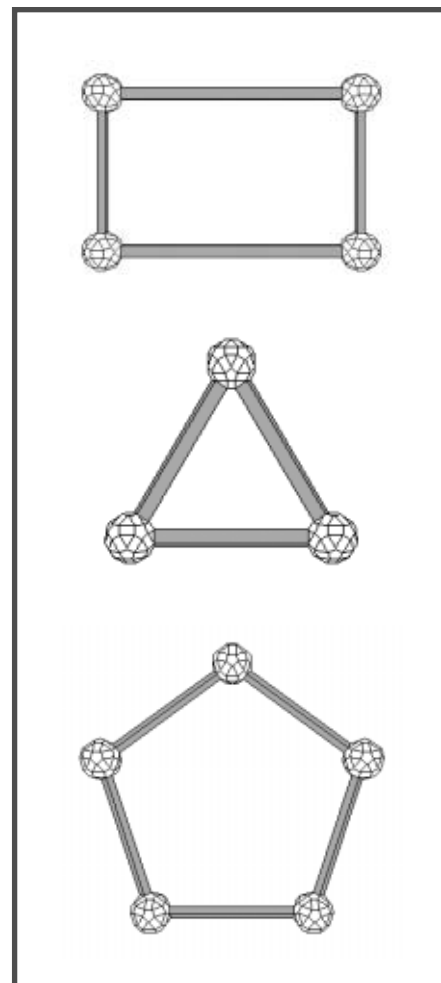
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Allow 5-10 minutes for students to build pincushions and count the number of struts. *What's the best way to count the struts?* While many students will want to remove the struts and count them, others will have discovered that when the pincushion is flat on a desk top, the struts can be counted using symmetry. For example, a yellow pincushion has 4 "levels," each containing 5 struts. It is now easy to see that there are 20 triangular holes. The blue pincushion is especially interesting since it can stand on either 3 or 5 struts.

Chart the students' results on the board.

Is there a relationship between the shape of the hole and the number of holes in the node? In the lesson "Shape and Number Students" students established that the rectangular hole is the number 2, the triangular hole is the number 3 and the pentagonal hole is the number 5. Lead your students to multiply the "shape" by the number of holes in the node. Students will discover that the result is always 60.

How can we get the number 60 from the numbers 2, 3 and 5? Lead your students to prime factors. *Are there any smaller whole numbers which multiply together to make 60?* Let your students explain why the number 1 doesn't count. *Where else can we find the number 60 in this classroom?* Someone will discover the clock. *Why are there 60 minutes in an hour?* Lead students to find how many ways 60 minutes can be



Shape	Number represented by shape	Holes per level	Number of levels	Subtotal	total holes
rectangle	2	5	4	20	30
		10	1	10	
triangle	3	5	4	20	20
pentagon	5	3	4	12	12
				grand total	62

divided into whole number increments (2, 3, 4, 5, 6, 10, 12, 15, 20, 30...) Ask the class to find other familiar numbers with 2, 3 and 5 as prime factors (24 hours in a day, 360 degrees in a circle)

As an extension activity students can explore how many lines pass through each node. Lead students to discover that each strut in the pincushion represents a vector originating at the point (node) in the center. Two 2 vectors (struts) point in opposite directions to form a line in space. The relationship between the shape of the strut (it's number) and the number of such lines passing through the node can be described with the diagram on page 106. To obtain the number of red, or number five lines, cover the 5 and multiply the remaining numbers ($3 \times 2 = 6$). As each line consists of two strut there are then 12 red holes in the node. Test that the system holds up for the other colors.

Assessment:

Observe students while they work, and question them about the method they used to count the number of holes in the node. To meet the standard students must use the symmetry method to determine that the nodes have 62 holes. To exceed the standard they must identify the number 2, 3, and 5 as being prime factors for 60.

Standards Addressed:

- * Mathematics standards addressing **mathematical problem solving as a method of inquiry and application** (NCTM Standard 1).
- * Mathematics standards addressing **number systems and number theory** (NCTM Standard 6).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Prime numbers and prime factors in common experience.

1	2	3	5	7	11	13	17
19	23	29	31	37	41		
43	47	53	57	59....			

Mathematics Intermediate Concept

Lesson Objective:

Students will be introduced to 2-fold, 3-fold and 5-fold symmetry embedded in Zome System's blue, yellow and red planes, respectively.

Prerequisite Skills:

Familiarity with Zome System and the concept of planes. Students also need to have experience relating geometric shapes and symmetries to numbers ("Shape and Number," "Similar Triangles," "What is Reflection Symmetry?" "Multiple Reflection Symmetry," "Translational Symmetry in Tilings", and "3-D Triangles").

Time Needed:

One class period of 45-60 minutes.

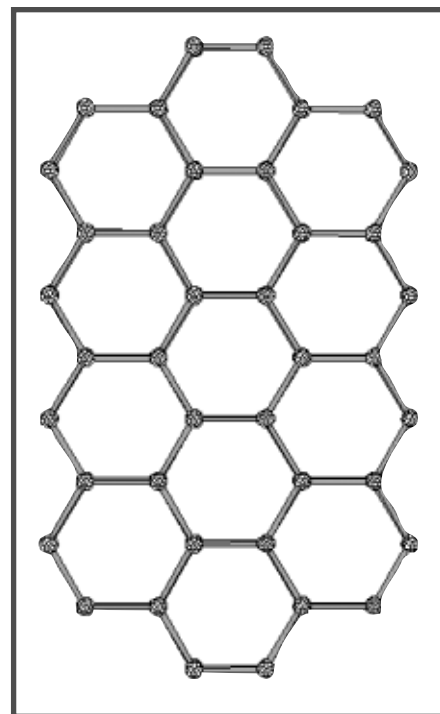
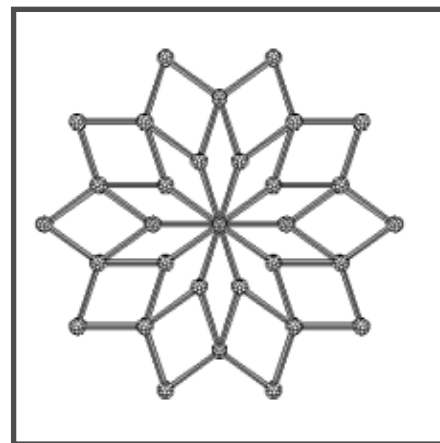
Materials Needed:

- Two Zome System Creator Kits for class of up to 32 students
- Overhead projector

Procedure:

Divide your class into teams of 4 students each and distribute Zome System components evenly among the teams. Challenge each team to build a "flat pattern" using Zome System. Explain that "flat" means all the nodes will touch the surface of the table the team is working around. This is a good opportunity to review the concept of a plane introduced in the "3-D Triangles" exploration. *Will any triangle built in Zome System lie flat on the table? What if another node is added to the triangular structure? Must all four nodes lie flat on the table? How many planes can pass through a triangle represented by a Zome System triangle? How many planes can pass through a line represented by a Zome System strut?*

How many planes pass through a 3-D triangle? You may also



wish to discuss the meaning of the word “pattern,” using examples from around your classroom: ceiling tiles, wallpaper patterns, graph paper, a group of marbles in the bottom of a bowl, etc.

Allow 10-15 minutes for the initial exploration, during which time you can offer guidance to teams on an individual basis. If any teams are having trouble getting started, suggest that they use one of the shapes they built during the “Shape and Number” exploration as a “seed” to start the pattern.

Patterns may be displayed with your overhead projector or hung with push-pins on a wall or chalkboard. Ask the teams if they can find a strut which fits into one of the holes in a node in their pattern so that it stands straight up (perpendicular) from the pattern, just as they did in the “Shape and Number” exploration. *How many different kinds of struts will stick straight up from a pattern* (All of the struts perpendicular to a given pattern will be the same color. In certain patterns, no strut will be perpendicular to the plane)? Hold a class discussion of the exploration. *How many different shapes of struts could stick straight up from a pattern? What does it mean means if no strut is perpendicular to the pattern? Are there parts missing from our Zome System kit? Is there a relationship between the pattern and the shape of the strut sticking up from the pattern?*

By the end of the period there will likely be consensus that the patterns perpendicular to the blue strut (in the blue plane) will exhibit symmetries related to the number 2, patterns perpendicular to the yellow strut (in the yellow plane) will exhibit symmetries related to the number 3, and patterns perpendicular to the red strut (in the red plane) will exhibit symmetries related to the number 5. Patterns without a perpendicular strut lie in the “green plane,” which is perpendicular to the available green lines/struts which can be purchased separately from the regular Zome System kits. Student should write their conclusions in their math journals.

Assessment:

Review notes in math journals, and take notes of discussions. To meet the standard students must complete a 2-D pattern. They must also determine that all the nodes in the structure are rotated in the same direction, so only one strut can be perpendicular to the pattern. To exceed the standard they must draw the conclusion that the perpendicular strut defines the symmetry number in the pattern.

Standards Addressed:

* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

More advanced work on tilings in 2 and 3 dimensions (“Non-Periodic Tilings - I: Kepler’s Tilings,” “Non-Periodic Tilings - II: Richert-Penrose Tilings,” “3-D Triangle Tiles,” “Plato’s Solids - I,” “Plato’s Solids - II,” and “Bridge Building Unit”).

Mathematics / Art Intermediate Concept

Lesson Objective:

Students will learn the concept of 2-dimensional translational symmetry by manipulating a flat grid structure in relation to a full-sized image, produced using sunlight.

Prerequisite Skills:

Familiarity with tessellations/tilings (“Trying Tessellation,” “Plane Patterns,” and “Triangle Tiles 1”). Previous work on symmetry concepts (“What is Reflection Symmetry?,” “Multiple Reflection Symmetry,” and “Rotational Symmetry”)

Time Needed:

One class period of 45 to 60 minutes.

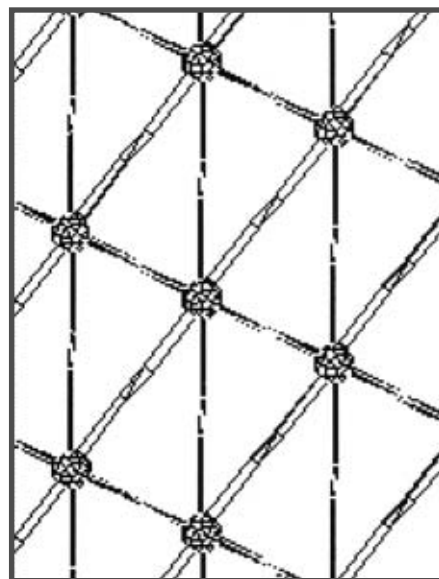
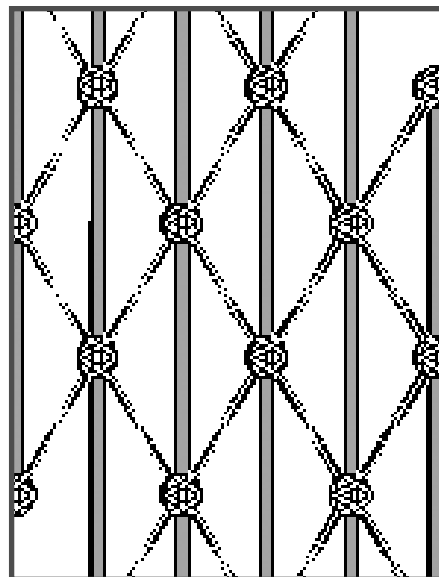
Materials Needed:

- Two Zome System Creator Kits for class of 25-30 students
- One 9”x12” sheet of blueprint paper per team of 3 students (see “Resources” section)
- One piece of 12”x 15” cardboard per team, to cast shadows upon
- One cardboard box per team, large enough to fit over Zome System model and sheet of blueprint paper
- Household ammonia solution
- One 9x12 plastic tub or cake pan (not aluminum)
- Masking tape

Procedure:

Prepare the supplies needed by following the instructions in the “Resources” document. The required ammonia solution can be harmful, the “Caution” and “First Aid” notices should be taken very seriously. Cut the large sheets of blueprint paper into 9”x12” sections.

Since students will be working with a photosensitive material, the work area should be set up in an area out of



Triangle Tiles - II

Zome System

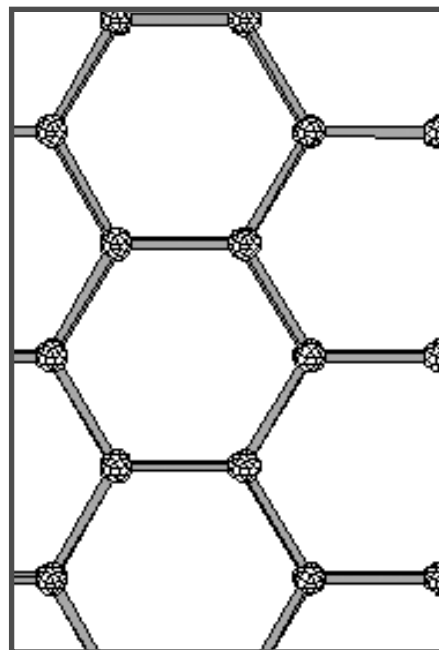
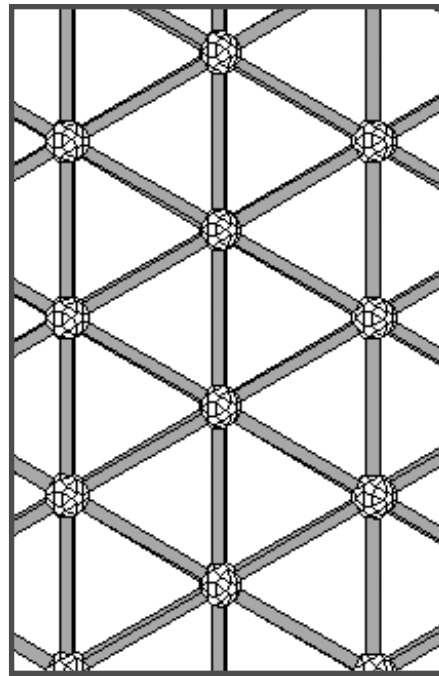
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direct sunlight. The paper will be exposed by direct sunlight in a few seconds, fluorescent light in less than an hour, incandescent light will take several hours.

The first task for your students will use the triangular flat grid structures they built in the "Triangle Tiles 1" class, is to make a full size "photo" of their finished pattern. Distribute the grids and a sheet of blueprint paper to each team. The teams should mount blueprint paper on cardboard, and attach their grids on top. The assemblies should be brought outdoors and placed on a flat, level surface when the sun is relatively high in the sky. Place weights on the corners of the paper to keep it from blowing away. While the "photos" are being exposed, discuss what is happening. *Why does the sun need to be high in the sky? Why does the paper & model assembly need to be flat and level? What is happening to the paper while we wait?* When the time is up, the teams should bring their assemblies back inside. Lay the paper flat on the table and the model flat on the paper so it lines up exactly with its image.

The second task for the teams is to find out how many directions they can move their model from its central position so that it lines up with its image again. Ask teams to discuss their findings and write their conclusions in their math journals. Follow up with a class discussion of the exploration. *How many directions can you move the pattern so that it lines up with its image again? Why? Did one team find more directions than another? Why, or why not? Do the patterns line up with their images if you move them by just one triangle? Two triangles? Three triangles? More? Can these kinds of patterns go on forever?*

A motif that can be shifted and repeated has **translational symmetry**. The resulting pattern is **periodic**, in that it is created by infinite repetition of the same motif. Ask students to identify examples of translational symmetry from the classroom or elsewhere (bricks, ceiling tiles, border decorations in books, parking meters in a row, soldiers standing in formation, etc.)



Zome System

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Triangle Tiles - II

Assessment:

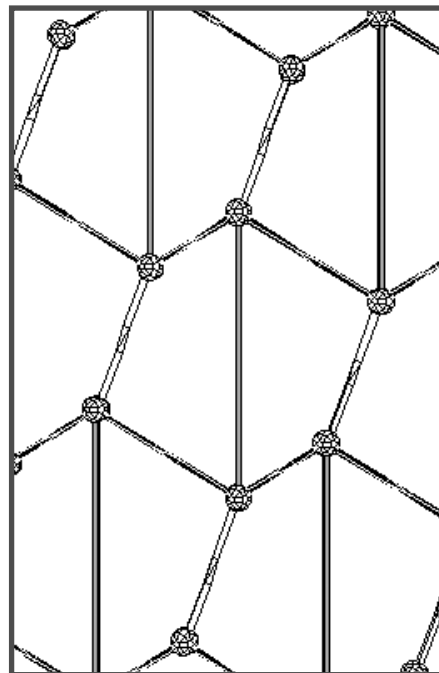
Question students while they are experimenting, and review their math journals. To meet the standard students must find the number of directions their pattern repeat in. To exceed the standard they must how the pattern is periodic and connect this type of symmetry to other occurrences of translation symmetry.

Standards Addressed:

- * Fine Arts standards that **identify and apply the elements of art in a variety of media.**
- * Mathematics standards addressing **mathematics as a means of communications** (NCTM Standard 2).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Work on advanced tilings (“Non-Periodic Tilings-I: Kepler’s Tilings,” and “Non-Periodic Tilings-II: Richert-Penrose Tilings”). Also valuable before working on projection modeling (“Cubes - I,” “Cubes - II,” “Cubes - III,” and “Cubes - IV”).



Zome System

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Tilings with Multiple Symmetries

Mathematics Intermediate Concept

Lesson Objective:

Students will understand how some tilings can display rotational and/or reflection symmetry in addition to translational symmetry. They will learn how to demonstrate how these tilings can be generated using either one of the symmetry concepts.

Prerequisite Skills:

Ability to differentiate between different types of symmetry ("What is Reflection Symmetry," "Multiple Reflection Symmetry," "Rotational Symmetry," "Plane Patterns," "Triangle Tiles - I," and "Triangle Tiles - II").

Time Needed:

One class period of 45-60 minutes.

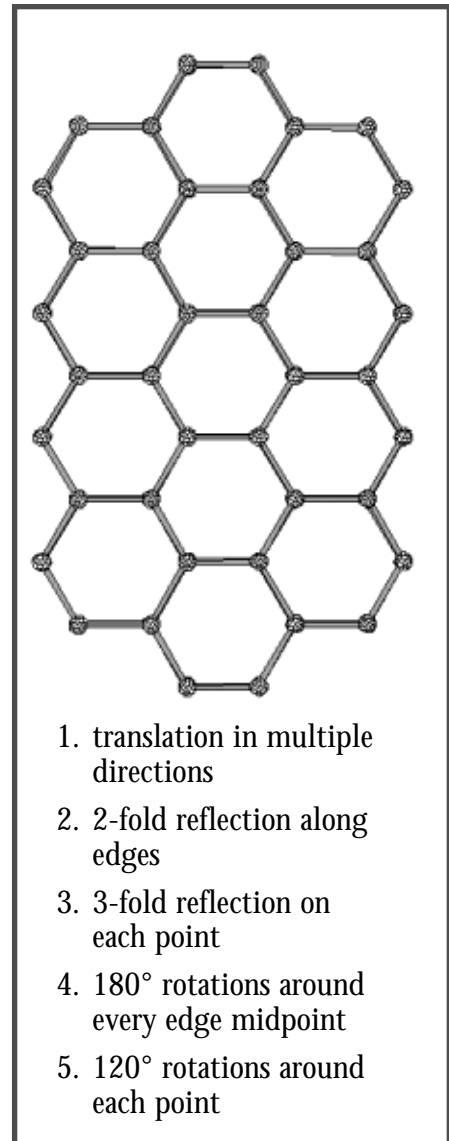
Materials Needed:

- Two Zome System Creator Kits for 25-30 students.

Procedure:

Start the class with a brief review of how tilings work. Students should restate their definitions of translation, rotation, and reflection symmetry. The following unit will cover tilings that display more than one type of symmetry, in addition to translational symmetry.

Divide the class into teams of 3-5 students, and distribute the Zome System pieces evenly. Each team is to build a polygon, and discover how to tile with it. Allow 15 to 20 minutes for this exploration. When the tiling are completed, students should study them and discuss their findings. *Does it have any symmetries? More than one? More than two? Which ones? What is the symmetry number (for example, 2-fold, 3-fold, and upwards) for each type of symmetry in their designs?* All findings should be recorded by the



Tilings with Multiple Symmetries

Zome System

Builds Genius!

students in their math journals.

The teams should introduce their tiling to the rest of the class. The presentation should include a sample of the tile used in the tiling, and which symmetries are inherent in the tiling. The team should demonstrate how the various forms of symmetry can be used to repeat the tile across the plane. *Have all the symmetries been found? Are there any others in the design?*

See graphics for sample symmetry analyses in tilings.

Assessment:

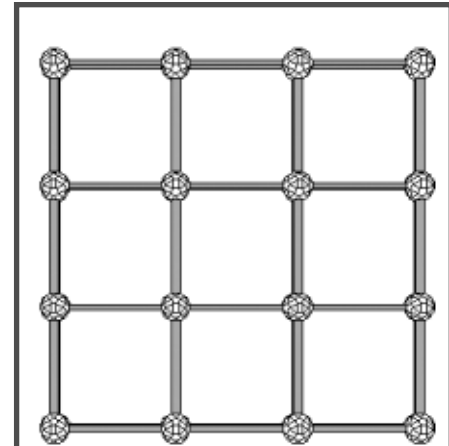
Discuss the definitions offered by the students individually and with the class, and review their math journals. To meet the standard, students must build a tiling with multiple symmetries, and be able to present these symmetries to the class. To exceed the standard they must verbalize definitions for the various symmetries.

Standards Addressed:

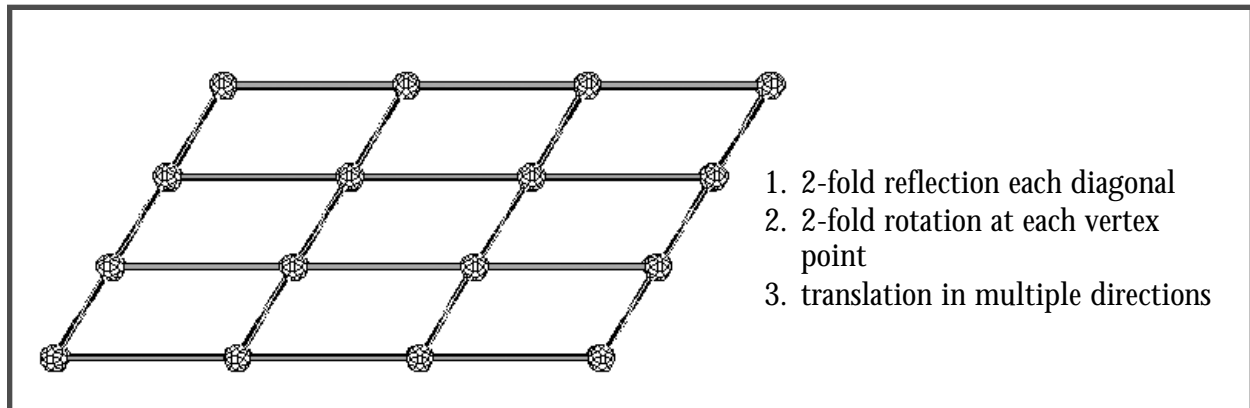
* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Further explorations of symmetry and tilings in 2 and 3 dimensions ("Non-Periodic Tilings - I: Kepler's Tilings," "Non-Periodic Tilings - II: Richert-Penrose Tilings," "Spiral Symmetries," and "3-D Triangle Tiles").



1. translation in multiple directions
2. 180° rotations around edge mid-points (2-fold rotation)
3. 90° rotations around each point (4-fold rotation)
4. 4-fold reflection at points
5. 2-fold reflection at edge midpoints



1. 2-fold reflection each diagonal
2. 2-fold rotation at each vertex point
3. translation in multiple directions

Mathematics Intermediate Concept

Lesson Objective:

Students will learn how combinations of rotation symmetry and translation symmetry can generate spiral symmetry in 3 dimensions.

Prerequisite Skills:

Ability to differentiate between different types of symmetry ("What is Reflection Symmetry," "Multiple Reflection Symmetry," "Rotational Symmetry," "Translational Symmetries in Tilings," "Tilings with Multiple Symmetries," "Plane Patterns," "Triangle Tiles - I," and "Triangle Tiles - II").

Time Needed:

One class period of 45-60 minutes.

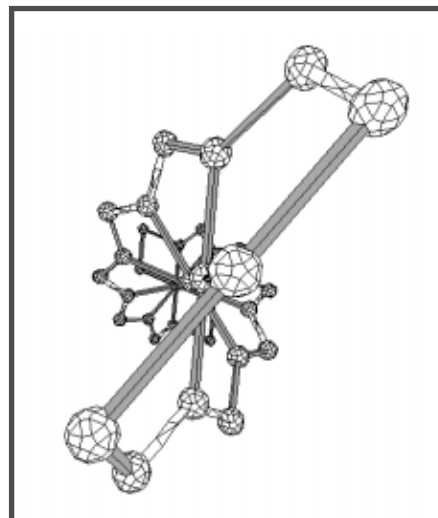
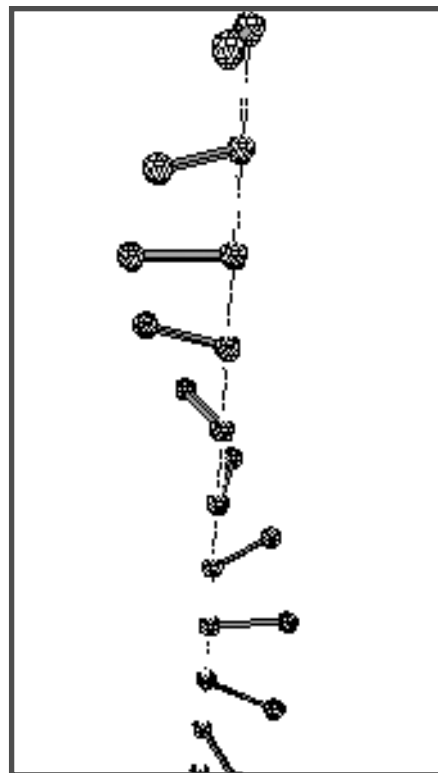
Materials Needed:

- Two Zome System Creator Kits for class of 25-30 students
- Objects, or pictures of objects, displaying spiral shapes.

Procedure:

Prepare for the class by building a Zome System spiral consisting of 10 short red struts in a line. On each node in the line put a short blue strut at right angles to the line. Each successive blue strut in the line should be rotated 36° relative to the previous blue strut. This will generate a spiral shape around the red line. The bottom node can have 5 equally spaced blue struts to form a base.

Inform the class that they are going to continue their exploration of symmetry concepts. Show them the model of the spiral described above, and initiate a short discussion about it. *What is the name of this type of shape (spiral)? Where do we encounter this shape in nature? How about in architecture, advertising, or other man-made objects?* Examples



Spiral Symmetry

Zome System

Builds Genius!

can include a DNA molecule, a water vortex (whirlpools), curved animal horns, sea shells, climbing vines, spiral staircases, candy canes, barber poles, metal springs, slinkys, wood screws, and so on. If available, let students study some objects, or images of objects, with spiral shapes. *Why do spirals occur in nature? Why are they useful to us. Why would we choose to build a spiral, as opposed to a regular, staircase? Why does a wood screw have a spiral pattern?*

Divide the class into teams of 3-4 students, and distribute the Zome System pieces evenly. Their first task is to either build a copy of your model, or design a spiral model of their own. While they build they should discuss what kind, or kinds, of symmetry, if any, the spiral has. They must make notes of their discoveries. Allow 15-20 minutes for this segment. Circulate and assist as necessary.

The teams should present their models to the rest of the class. *What kind(s) of symmetry can they find in the model? Can they have a combination of more than one type of symmetry?* List ideas on the board. Discuss the list until the students realize that both translation and rotation symmetries are needed to generate the spiral symmetry in their models. For instance, in the demonstration model, each new position on the spiral is generated by a translation along one short red strut, followed by a rotation of 36° around the axis. Contrast this combination of symmetries with those that can be found in tilings (“Tilings and Multiple Symmetries”).

Close the class by asking each student to give a written definition of spiral symmetry. Their report should show how the symmetries are displayed by the model they built during the class.

Assessment:

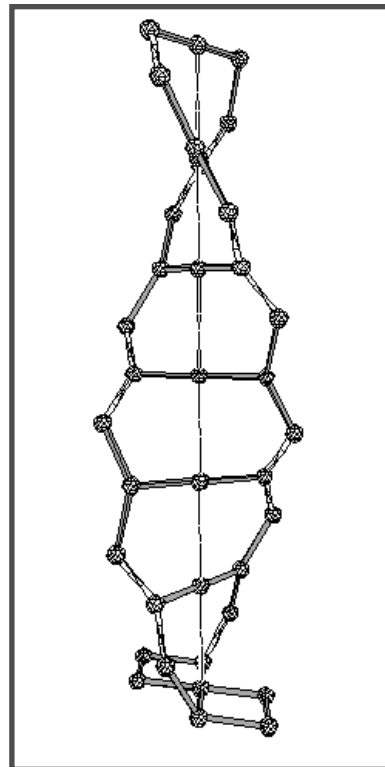
Observe the teams while they experiment, and review notes in their math journals. To meet the standard the students must design a spiral model, and show that is symmetrical. To exceed the standard they must write an accurate definition of the spiral symmetry in their model.

Standards Addressed:

* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Further explorations of symmetry and spirals in 2 and 3 dimensions (“Finding Tau”). Discussion of the use of spirals in architecture, design, engineering, biology, etc.



Mathematics Intermediate Concept

Lesson Objective:

Students will analyze the symmetries of quadrilateral tilings. They will discover how various types of symmetries, used alone or in combination, can generate the same pattern more than one way.

Prerequisite Skills:

Some previous work with tessellation (“Trying Tessellation,” “What are Quadrilaterals” “Tiling with Quadrilaterals,” and “Plane Patterns”). Basic understanding of symmetry concepts (“What is Reflection Symmetry?” “Multiple Reflection Symmetry,” “Rotational Symmetry.” and “Translational Symmetries in Tilings”).

Time Needed:

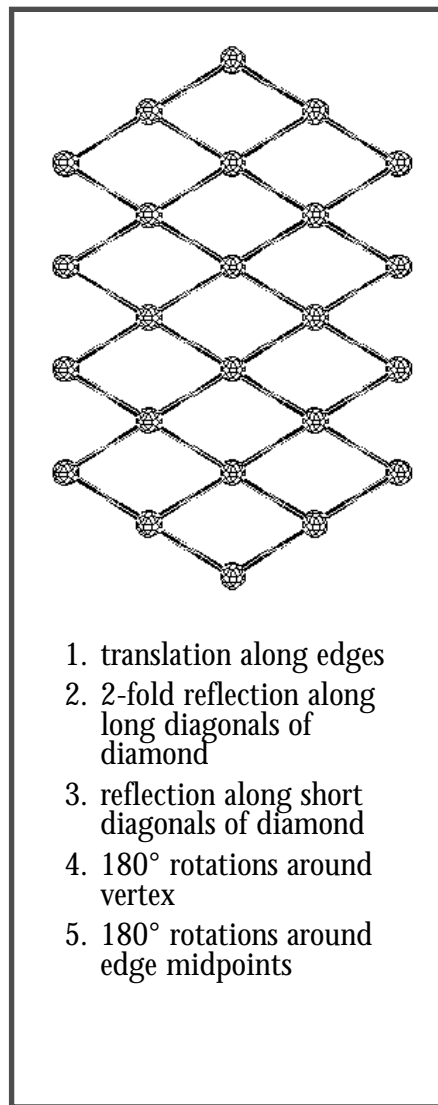
One class period of 45-60 minutes.

Materials Needed:

- Two Zome System Creator Kits for a class of 25-30 students
- Tilings from “Tilings with Quadrilaterals?” lesson

Procedure:

Review with the students the basic types of **symmetry**; **translation**, **reflection**, and **rotation**. Regroup students into their teams from the “Tilings with Quadrilateral” lesson, and return their tilings, or trade among teams. Challenge the students to determine in their groups how many different ways they can generate the tiling, identically to the original, using the various forms of symmetry they have learned. *Can the patterns be generated with a combination of symmetries? How many forms of symmetry can be used?* Allow 10-15 minutes for the teams to discuss their solutions and possibilities. Ask each team to explain their



Symmetries in Quadrilateral Tilings

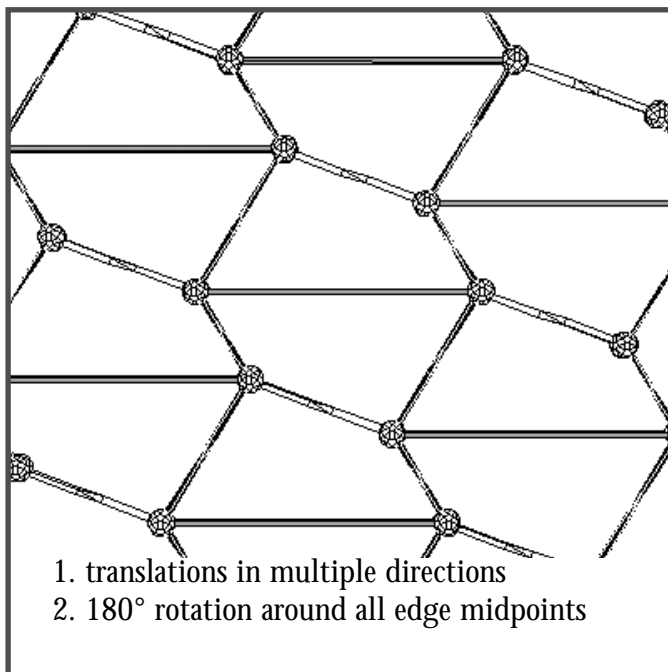
solutions to the rest of the class, using an individual tile to demonstrate how each form of symmetry can translate the tile to anywhere in the overall pattern. List the different types of symmetry on the board as they are discussed. Examples of various types of quadrilaterals are shown here along with an analysis of their respective symmetries.

Standards Addressed:

* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

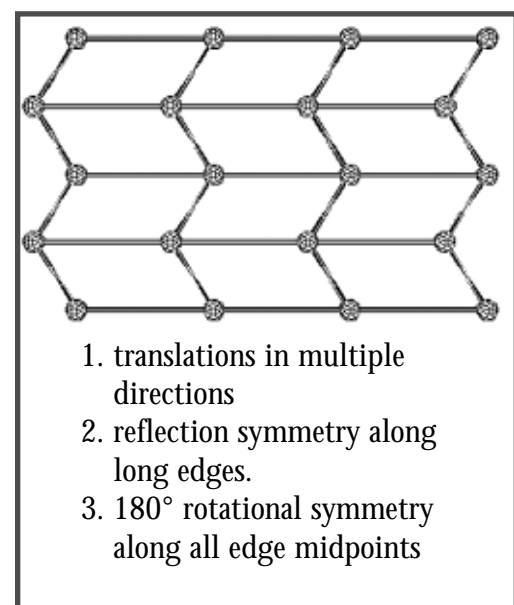
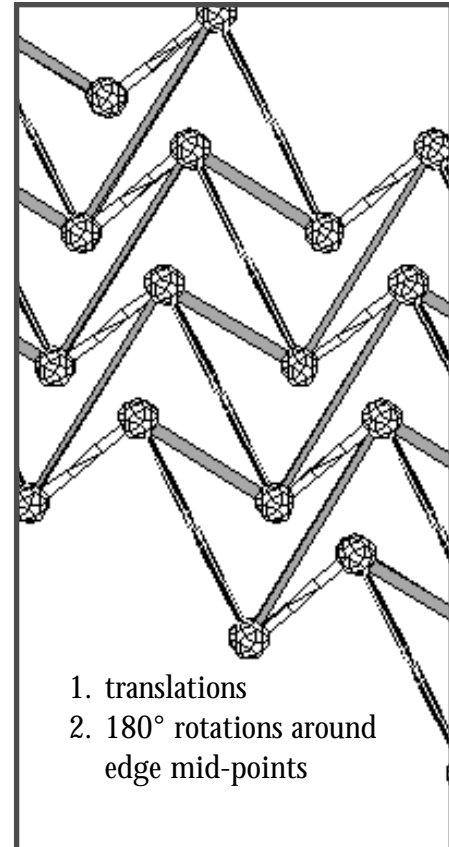
Transfer Possibilities:

More advanced tilings in two and three dimensions ("Non-Periodic Tilings-I: Kepler's Tilings," "Non-Periodic Tilings-II: Richert-Penrose Tilings," "3-D Triangles," "3-D Triangle Tiles," "Plato's Solids - I," and "Plato's Solids - II").



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Measures of Space - I: Lengths and Areas

Mathematics Intermediate Concept

Lesson Objective:

Students will learn how to measure in one and two dimensional space. They will calculate how much space any object of a given dimension occupies, based on its measurements.

Prerequisite Skills:

Experience working with 2 and 3 dimensions ("2-D and 3-D Shapes," and "Speed Lines!").

Time Needed:

One or two class periods of 45-60 minutes.

Materials Needed:

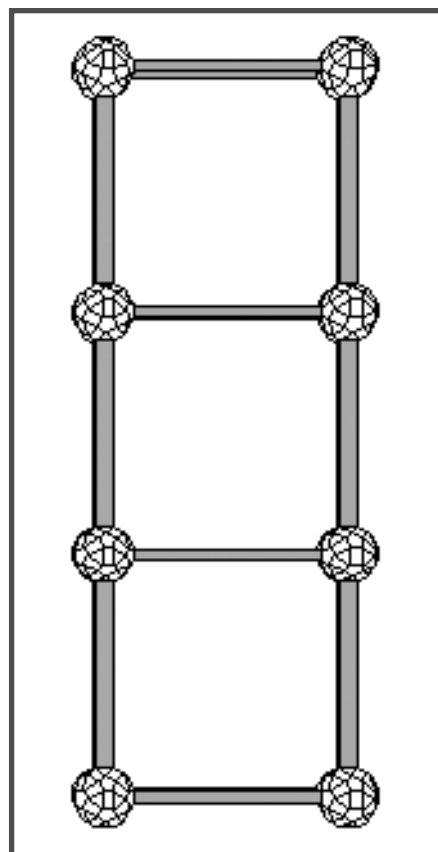
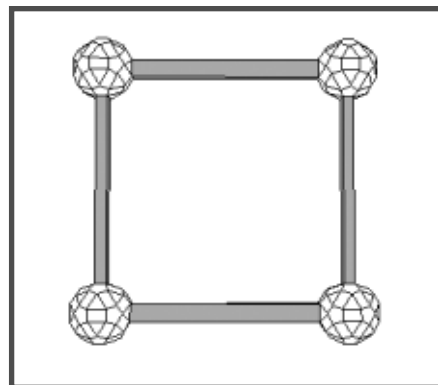
- Two Zome System Creator Kits for 25-30 students

Procedure:

Start the class with a discussion about dimensions. *What are dimensions? What is the difference between objects with 2 dimensions, and those with 3 dimensions? What does a 1-dimensional object look like?* Allow the students to debate until they have agreed that a line is the representation of 1-D space. *How many directions can we move in in 3 dimensions? In 2 dimensions? In 1 dimension? What can we measure in 1 dimension?* The students should try to agree on definitions and enter them in their math journals. A formal definition might state that; A **line** is a space of one dimension. In this space we can move right (by convention known as the positive direction) or left (negative direction). We can not move in (positive) or out (negative), and we can not move up (positive) or down (negative), outside of the line in one dimension.

A **length** on a line is a portion of the line which can be measured in units of our choosing.

Divide the class into teams of 3-4 students, and dis-



Measures of Space - I: Lengths and Areas

tribute the Zome System pieces. The first project for the teams is to build a 1-D structure using blue lines and determine a useful system to measure it. They are to present the structure to the class and report its measurements. No more information than this should be given.

The students will rapidly figure out that the structure is just a continuous line of blue struts connected by nodes. They are likely to spend a few minutes more agreeing on a unit of measurement. As they are not allowed to use a traditional measuring instruments, they will have to settle on one of the 3 strut lengths as their unit. In order to report a measurement, they can only use struts of their chosen unit. *What is their chosen unit? How long is their line? What happens when we subtract 2 units from this? What is left over? What is the benefit of agreeing on units of measurement in this fashion?* From the discussion, the class will discover that the lengths can added or subtracted just like numbers.

The next step is to add another dimension. *What would be the result if we were also able to move up and down as well as right and left? How will this change the teams' structures? If one strut is a unit piece of one dimension space, then what is a unit piece of two dimension space (square of same size struts)?* Allow the teams 5-10 minutes more to build their resulting square grids and settle on the square they will use as a measuring unit in 2-dimensional space.

After discussing the 2-D structures, the students should agree on a definition of **area**, and write it in their math journals. A formal definition might state that; Area is two dimensional space, or a plane, which can be measured in square units of our choosing. *Is there a way to determine the number of squares in a structure by only counting some of them* (Yes, if the overall shape is a rectangle, we can multiply the lengths of the two sides)?

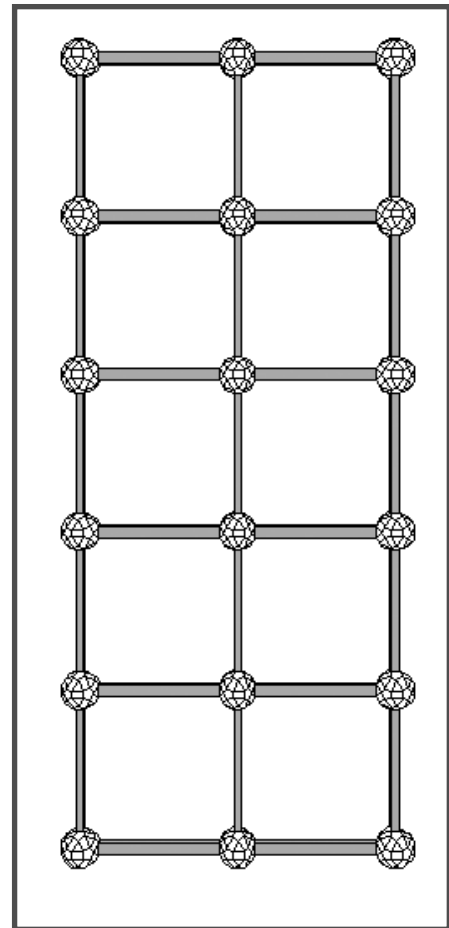
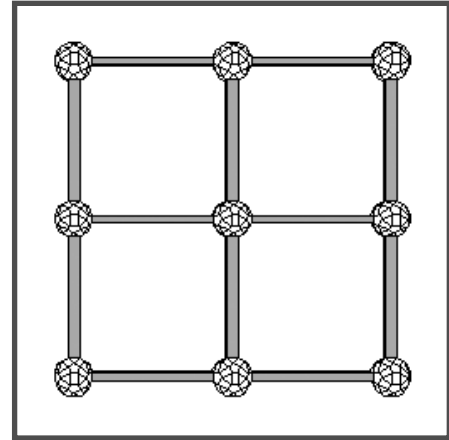
Save the grids for the lesson "Measures of Space - II: Volumes".

Assessment:

Take notes while the students discuss their structures and definitions. Review notes in math journals. To meet the standard students must build the line and the grid and

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Measures of Space - I: Lengths and Areas

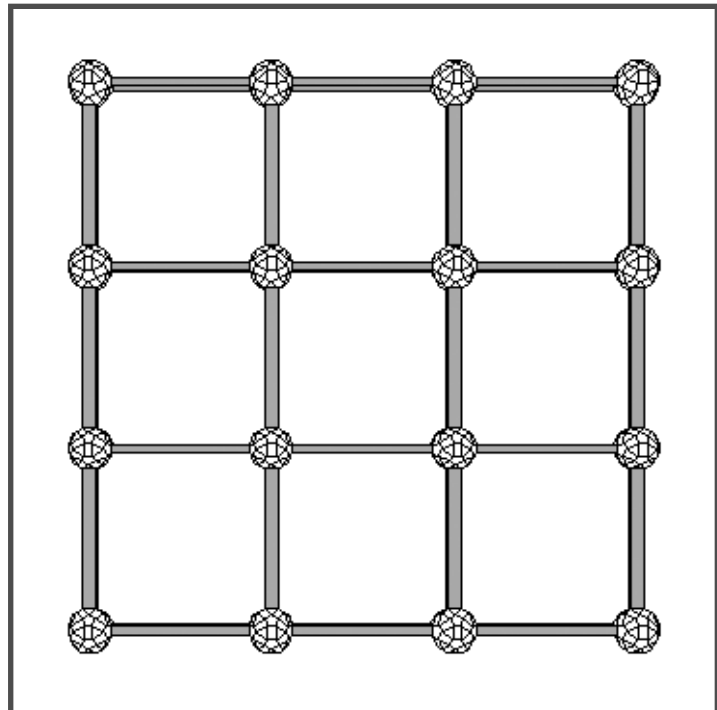
show that they understand the measurements in two and three dimensions. To exceed the standard they must verbalize correct definitions for line, and for area.

Standards Addressed:

- * Mathematics standards addressing **mathematics as a means of communications** (NCTM Standard 2).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).
- * Mathematics standards addressing extensive **concrete experiences using measurement** (NCTM Standard 13).

Transfer Possibilities:

Study of area of various shapes, and measurements of volume ("Measures of Space - II: Volumes"). Discussions of standardization of measuring units.



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Measures of Space - II: Volume

Mathematics / Biology Intermediate Concept

Lesson Objective:

Students will explore **volume** and measurement of three dimensional (3-D) space. They will determine that volume increases faster than surface area if one proportionally increases all the linear dimensions of an object.

Prerequisite Skills:

Experience working with 2 and 3 dimensions ("2-D and 3-D shapes," "Speed Lines!" and "Measures of Space - I: Lengths and Areas"). Some previous exposure to concepts of volume ("Volume for Beginners").

Time Needed:

One or two class periods of 45-60 minutes.

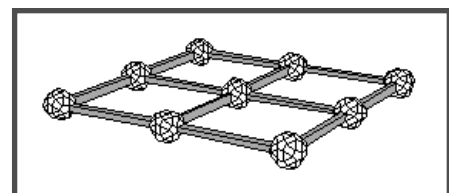
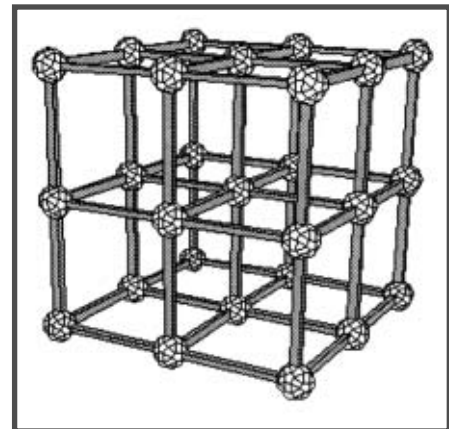
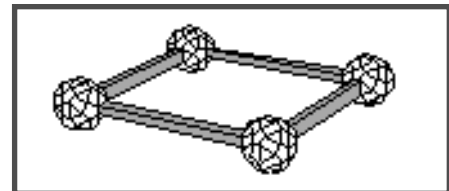
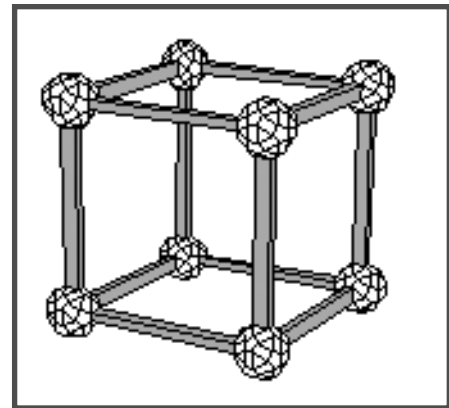
Materials Needed:

- Two Zome System Creator Kits for 25-30 students

Procedure:

Start the lesson with a brief review of the concepts involved in measuring 1-, and 2-dimensional space. *What is 1-dimensional space? What is length? What unit of measurement represents 1-D space in Zome System? What is 2-dimensional space? What is area? What unit of measurement represents 2-D space in Zome System?*

Volume is a natural extension of length and area, just a dimension higher. In this lesson students are going to explore measurements in 3-dimensional space. Divide the students into the teams from the lesson "Measures of Space - I: Lengths and Areas", return the grids built in that lesson, along with the remaining Zome System pieces. Their task is to add a third dimension to their grids and determine an appropriate unit of measurement for 3-D space. They must also report the resulting volume of their structure. *What kind of grid will result if we can move*



Measures of Space - II: Volume

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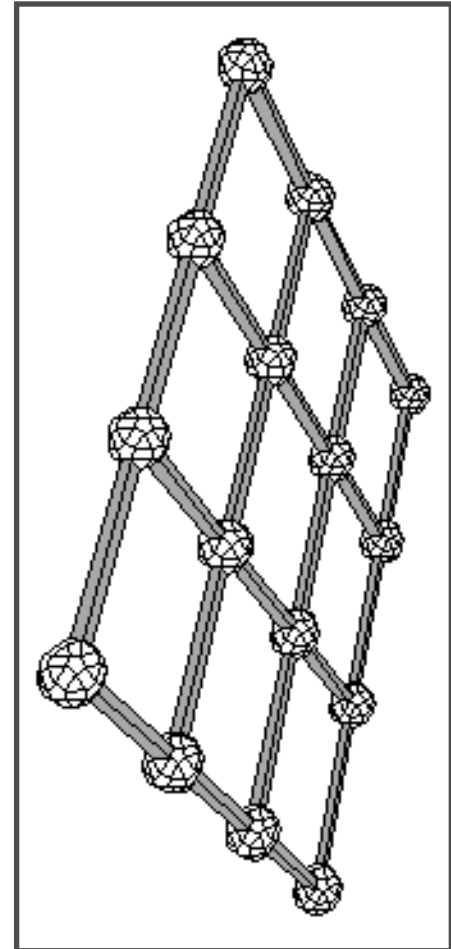
up and down as well? What is the unit of measurement of such a grid (a cube)? Allow the teams to build, and present configurations of these cubes such as a $2 \times 2 \times 2$ cube, a $3 \times 3 \times 3$ cube, and a $2 \times 3 \times 5$ box. What is the volume of the structure? What do we mean by volume? How much 3-D space in cube units do they occupy? A $2 \times 2 \times 2$ cube has 2^3 , or 8 cubic units, a $3 \times 3 \times 3$ cube has 3^3 , or 27 cubic units, and a $2 \times 3 \times 5$ cube has 30 cubic units.

Next, show how volume increases faster than area. Ask a third of the teams to build a 1×1 square, a third to build a 2×2 square, and a third a 3×3 square. What is the area of these grids? Have them calculate the resulting volume when the third dimension is added ($1 \times 1 \times 1$, $2 \times 2 \times 2$, and $3 \times 3 \times 3$ cubes). Write down the results in a table on the board. If we compare the rate of growth of the volume to the areas we can see that the volume grows to the 3rd power of the linear dimension whereas the area grows to the square of the linear dimension. What are some practical implications of these growth patterns?

An interesting example involves the possible size of animals. Let us suppose that the $1 \times 1 \times 1$ cube represents a 200 lb. mountain lion, and the 1×1 square represents the total area of her paws (1 sq. ft.) This means that the gravitational load on her paws equals 200 lb./sq. foot. What happens if we double her linear dimensions? The load then is 1600 lb./4 sq. feet = 400 lb./sq. foot. What happens if we triple her linear dimensions? The load becomes 5400 lb./9 sq. feet = 600 lb./sq. foot. How would this affect her ability to hunt? This type of volume to surface area relationship determines what size different types of animals can be, why an elephant has such large feet and legs in relation to its body, and why an insect can breathe through its skin, whereas a larger animal needs lungs. Why can a whale become so much larger than a land-based animal? What would human bodies look like if we only were 1 foot tall?

Assessment:

Give a short problem similar to the mountain lion example, and let students solve with or without the use of



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Measures of Space - II: Volume

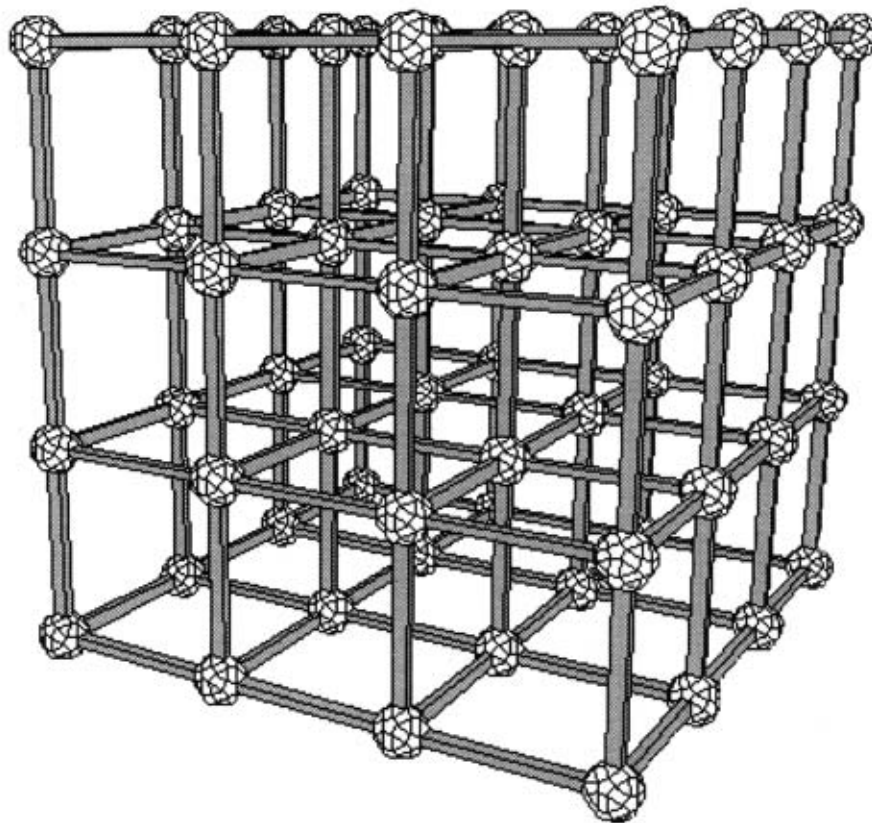
Zome System. Review their notes in their journals. To meet the standard students must build the cubic grid and be able to calculate its volume. To exceed the standard they must define the comparative growth rates of a square area and a cubic volume.

Standards Addressed:

- * Mathematics standards addressing **mathematics as a means of communications** (NCTM Standard 2).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).
- * Mathematics standards addressing **extensive concrete experiences using measurement** (NCTM Standard 13).

Transfer Possibilities:

Exploration of volume in non-cubic structures.



Mathematics Intermediate Concept

Lesson Objective:

Students will be exposed to the concepts of **point**, **line**, **plane** and **dimensions** in relationship to the triangle.

Prerequisite Skills:

Familiarity with Zome System and the concept of triangles (“Try the Triangle” and “Similar Triangles”). Ability to relate geometric figures to representative numbers (“Shape and Number”).

Time Needed:

One class period of 45-60 minutes.

Materials Needed:

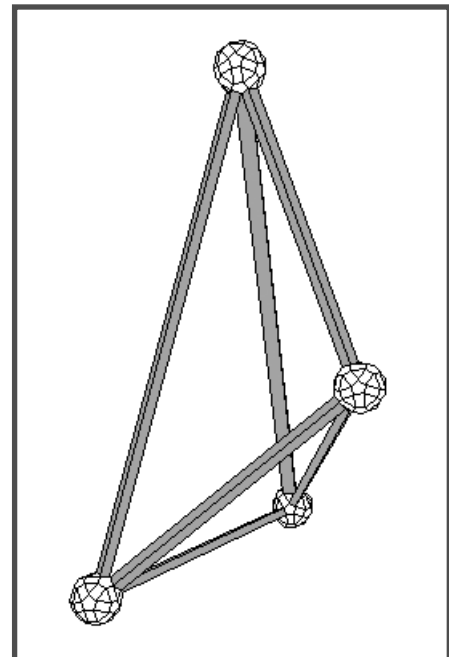
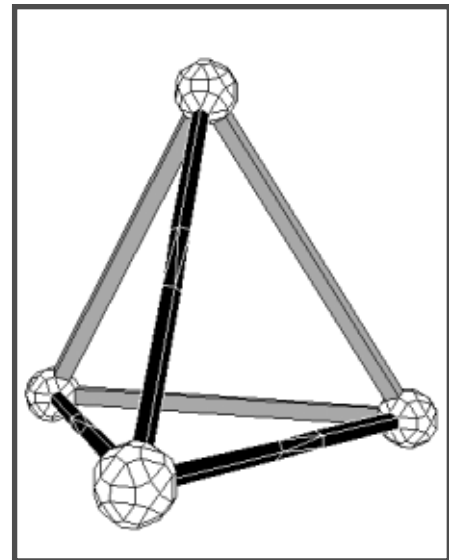
- Two Zome System Creator Kits for class of 25-30 students
- Overhead projector

Procedure:

Divide the class into teams of four students each and distribute Zome System components evenly among the teams. It is important that the teams start with the same number of components.

The team challenge is to build a “3-dimensional triangle.” Students may ask what you mean by a “3-dimensional triangle.” Rather than leading a group discussion at this point, suggest that team members discuss the concept among themselves as they each try to build a model. Allow at least 15 minutes for this exploration, during which time you can offer guidance to teams and students on an individual basis.

At the end of the exploration, ask a representative of each team to introduce the team’s “3-dimensional triangles” to the class and explain why it answers the challenge. Team representatives can use the overhead projector



3-D Triangles

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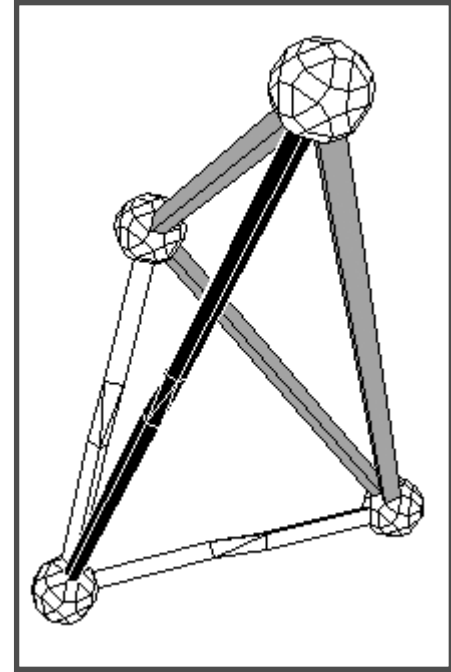
tor to show a 2-dimensional shadow of their 3-dimensional triangles.

After each presentation, involve the class in a discussion of the various concepts comprising the idea of a 3-dimensional triangle. *How do you know it is a 3-dimensional triangle? What do we mean by "dimension"? How many dimensions does a normal triangle have? Is a Zome System triangle really 2-dimensional? Why? Is a Zome System 3-dimensional triangle really 3-dimensional? Why? What do Zome System parts really represent? How many parts does it take to make a normal triangle? How many parts does it take to make a 3-dimensional triangle? Are all 2-dimensional triangles the same? In what ways? In what ways are they different? Are all 3-dimensional triangles the same? In what ways? In what ways are they different?*

Discussion of Zome System nodes and struts representing points and lines may also lead to philosophical questions such as: *Do points and lines really exist? Can anything exist in other than 3 dimensions? Is a piece of paper 2-dimensional? A shadow?* Finally you may wish to discuss the idea that the 3 points of any triangle define a plane.

As the students discuss what factors all 2-dimensional triangles have in common, and what factors all 3-dimensional triangles have in common, you may wish to chart the number relationships.

The table will help the students better understand the relationship between shapes and numbers introduced in



	Number of points	Number of lines	Number of 2-D triangles	Number of 3-D triangles
0-D triangles				
1-D triangles				
2-D triangles				
3-D triangles				
4-D triangles				

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3-D Triangles

the “Shape and Number” lesson plan. Direct students to record their observations (including this chart if applicable) in their math journals.

As an extension you can continue the discussion by asking what a “4-dimensional triangle” looks like? Is there any way to predict what factors are common in all “4-dimensional triangles,” based on the number chart on page 130?

Assessment:

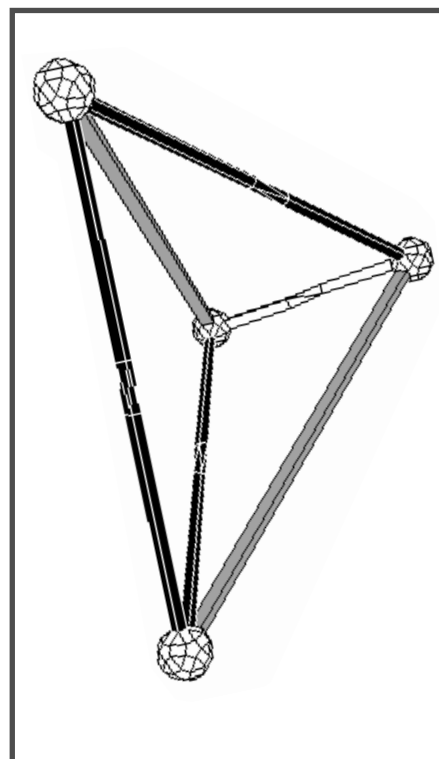
Take notes during the discussion, and study students’ written work in math logs. To meet the standard students must demonstrate understanding of the concepts of dimensions, points, lines, and planes. To exceed the standard, they must be able to explain the difference between the second, third, and fourth dimensions.

Standards Addressed:

- * Mathematics standards addressing **mathematics as a means of communications** (NCTM Standard 2).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

This lesson provides a good base for exploration of the use of geometry in architecture and structural engineering (“Tallest Tower in the World,” “Bridge Building Unit,” and “Space Frame Structures”).



Mathematics Intermediate Concept

Lesson Objective:

Students will apply concepts learned in the “Triangle Tiles” exploration to build a 3-dimensional triangular pattern called a space-frame structure.

Prerequisite skills:

Ability to define the properties of triangles and 2-, and 3-dimensional triangular patterns (“Shape and Number”, “Similar Triangles”, “3-Dimensional Triangles”, “Plane Patterns” and “Triangle Patterns”).

Time Needed:

One class period of 45-60 minutes.

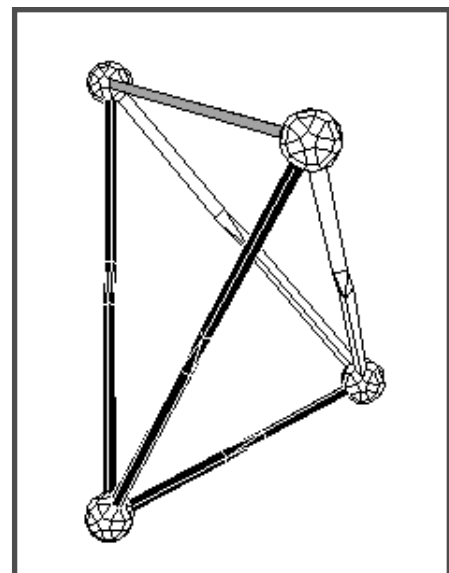
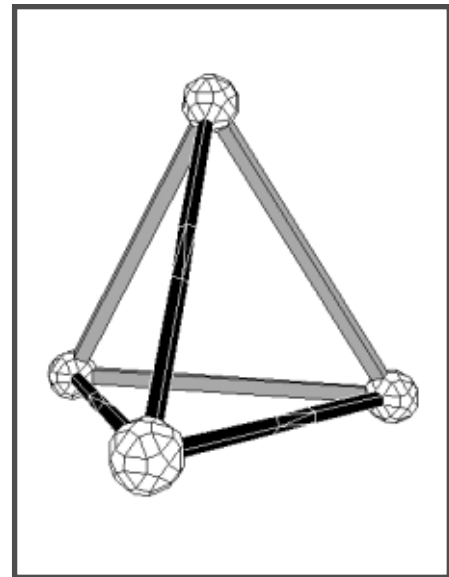
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Images of natural and man-made space-frame structures (These are useful for discussion purposes, and can include cross-sections of bone, cell structures of plants, bridges, high-voltage wire towers, architectural space frame structures, etc.)

Procedure:

Divide your class into teams of 4 students each and distribute Zome System components evenly among the teams. As in the “Triangle Tiles” exploration, it’s important that each team starts with the same number of each Zome System component. Set aside any remainders.

Based on their experience in the “3-D Triangles” and “Triangle Tiles” exploration, challenge each team to build a 3-dimensional triangular pattern; i.e. a 3-D pattern based on one unique 3-D triangle repeated many times. Again, the ultimate challenge will be to build the structure containing the most 3-D triangles. Inter-team trading of parts is allowed.



3-D Triangle Tiles

Zome System

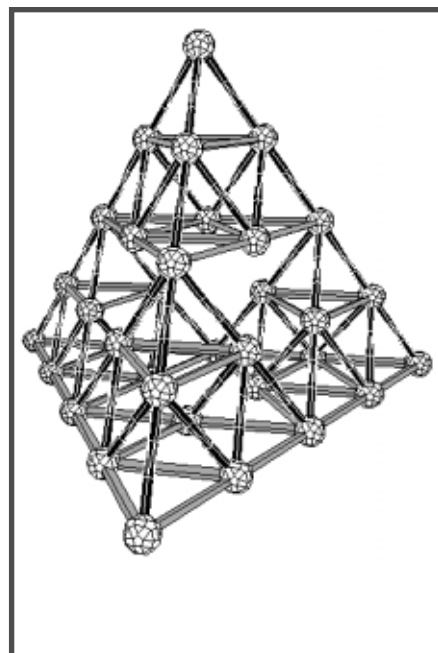
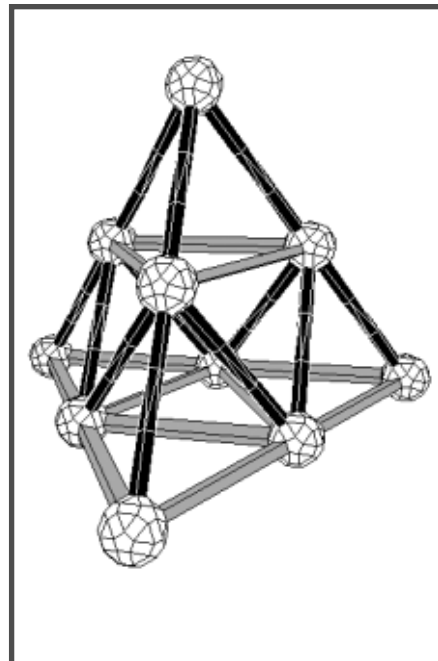
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Allow 10-15 minutes for this exploration, during which time you can offer guidance to teams on an individual basis. Teams can adapt the procedure used to build triangle patterns to the 3-D triangle tiles. Team members should first agree on a 3-D triangular “seed” which will be used as the basis for all cells in the pattern. Following this, each team member should make an exact copy of the seed triangle (for at least 4 total.) Teams may have to trade parts to build 4 units. The team must now figure out how to fit their 3-D triangles together to form a repeating pattern. This may be done by removing one or more nodes and/or struts from a given 3-D triangle, attaching it to the 3-D seed triangle and repeating the process until finding a pattern that works. Hint: one way to fit the 4 3-D triangles together is to build one larger 3-D triangle in which each edge is twice as long as the seed 3-D triangle. Finally, team members may expand their 3-dimensional tiling by copying the pattern and adding to the faces of the structure.

At the end of the building time have teams bring their models to the front of the class and discuss the strategy they used to maximize the number of 3-D triangles. Questions can include: *What factors affect the number of 3-D triangles you can build? The color of the struts? The number of identical struts in each one? The types of 3-D triangles used by other teams? How is the outcome affected if one team “copies” the structure built by another team? Should you make more long struts by adding together a short and a medium with a node in the middle? How does that affect the node supply?*

Follow up with a class discussion of the structures themselves: *How are the 3-D triangle patterns similar to the flat triangle tiles? How are they different? Are the 3-D triangular patterns made up only of 3-D triangles? If not, can anyone find a shape in the pattern which is not a 3-D triangle? How many sides does it have? Do 3-D triangular patterns have **translational symmetry** similar to 2-D triangle tile patterns? In how many directions? What structures in nature and the built environment have a similar pattern? Why?*

Ask teams to discuss their findings and write conclusions in their math journals. You may wish to introduce



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3-D Triangle Tiles

new vocabulary: a 3-D triangle is usually called a **tetrahedron** (meaning 4-faces). The 8-sided figure in between the tetrahedra is called an **octahedron** (meaning 8-faces). **Geodesic dome** inventor **R. Buckminster Fuller** coined the term **oct-tet truss** for these 3-D patterns, but in general they are referred to as space-frame structures.

Assessment:

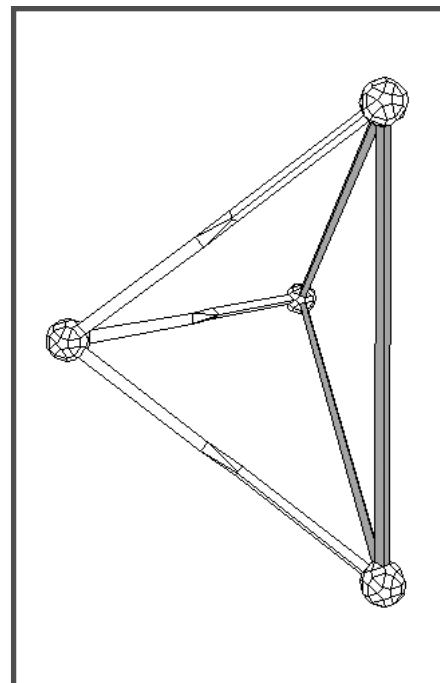
Review written work in math journals. Question students in group discussion using questions similar to those suggested in the procedure section. To meet the standard, students will be able to translate 2-D triangle tiles into 3-D triangle tiles. To exceed the standard, students will be able to identify the 3-D structures by name.

Standards Addressed:

- * Mathematics standards addressing mathematical problem solving as a method of inquiry and application (NCTM Standard 1).
- * Mathematics standards addressing mathematics as a means of communications (NCTM Standard 2).
- * Mathematics standards addressing investigation of mathematical connections (NCTM Standard 4).
- * Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Continued work on the use of geometry in architecture and structural engineering (“Livable City” and “Bridge Building Unit”).



Mathematics Intermediate Concept

Lesson Objective:

Students find and explore the Tau notation, and start working on the uses and occurrences of the Golden Section in mathematics and nature.

Prerequisite Skills:

Knowledge of basic polygon shapes (“Geometric Shapes”). Previous work on irrational numbers like pi is helpful.

Time Needed:

One class period of 45 to 60 minutes.

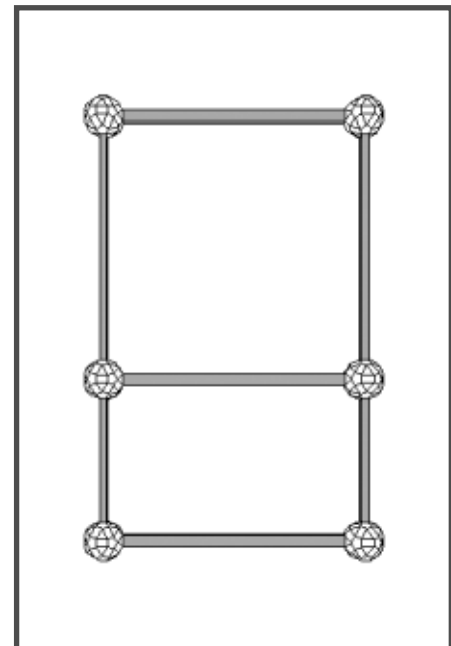
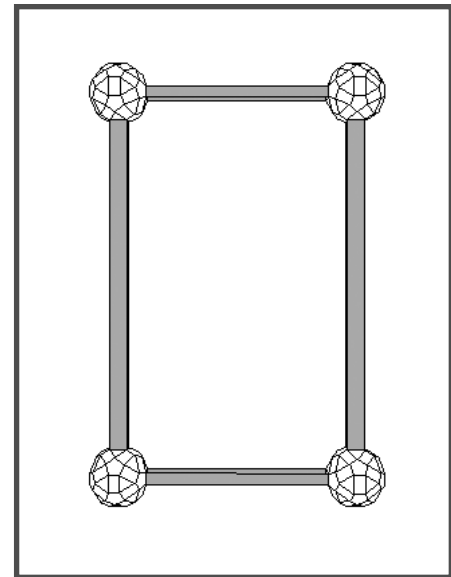
Materials Needed:

- One or two Zome System Creator Kits for 25-30 students
- One metric ruler, or rulers with inches divided into tenths, per team
- One calculator per team

Procedure:

Divide the class into teams of 2-3 students, and distribute Zome System pieces, rulers, and calculators. Introduce the topic by asking a few students to draw a rectangle on the board, and measure the height and width. Determine the ratio of the long side to the short side for each rectangle. Compare each ratio to 1.6, and explain that this is a number relationship known as the Golden Section, Golden Ratio, or Divine Proportion.

Their first task is to build a rectangle of the 2 short and 2 medium blue struts. *Is this a short, fat rectangle, or is it long and skinny? Have you seen this specific rectangle before?* The two different sides of this rectangle are in a special ratio called the Golden Section, represented by τ (the Greek letter tau). *What is a ratio? How do we describe or measure a ratio?* The sides are in the ratio $1:\tau$, where the



Finding Tau

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short side is 1 unit and the long side is τ units. *How can we find tau? What is the best way to measure this rectangle? Should the width of the nodes be included in the measurement?*

Ask the students to determine the ratio of the two sides of the rectangle. Representatives for a few teams can report their answer to the class. *Have all the teams got the same answer? What strategy did they use to determine the ratio?* To find the ratio, the teams should have divided the two edge lengths. For instance: $4.80/2.95$ inches = 1.627.

In what unit is τ measured (a ratio does not have a unit)? Next, add a square to the rectangle, as shown in the graphic. Measure the new side, and write it down. *What is the ratio* (apx. 7.75 inches / 4.80 inches = 1.614)?

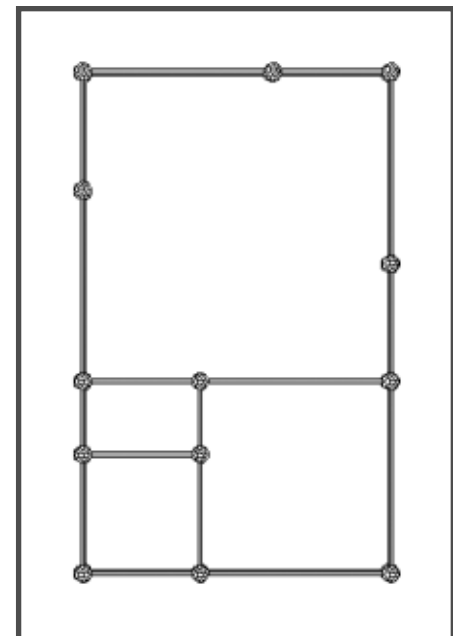
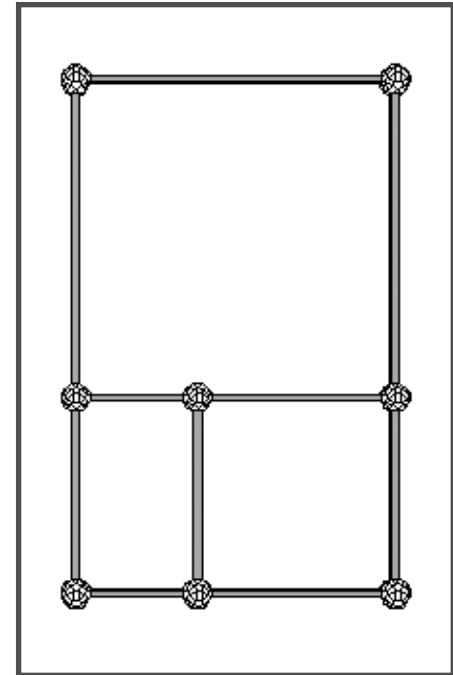
Record this ratio. *How close is it to the previous ratio?* This close ratio of sides tells us that our new rectangle is **similar** (same shape, different size) to the original one. Ask the teams to continue this process two more times, adding a square to the longer side of newly created rectangle. They should create a table to keep track of the ratios. *What is special with the Golden Rectangle? How do the rectangle and square interact with each other? What are the new lengths* (7.75, 12.55)? *What is the new ratio* (2.55 inches / 7.75 inches = 1.619)? *Can we continue adding squares? How?*

Bring the class back together and let them report their recorded ratios and any other findings. *How can we find an average ratio for all our measured ratios?*

The name of this ratio is tau, or τ . The actual value of tau is 1.6180339.... Like pi, the constant used in measuring circles and spheres, tau continues indefinitely, and its pattern doesn't repeat. This kind of number is called **irrational**. Each length we measured was longer than the one before it by 1.6180339... times. If our first length is 1 then the second is $\tau = 1.618$.

Tau appears in countless places, usually as a ratio between two lengths. For example, most temples in ancient Greece, including the Parthenon in Athens, were built in a rectangular shape with the exact same proportion as the rectangle the students just made.

The tau ratio also occurs in many natural objects,



including the human hand. The next task is for the teams to determine the ratios between their finger bones. Using rulers, they should measure the length of the first section of their fingers (fingertip to first joint), and write down that measurement. Continue measuring each section of the finger, and then the large knuckle to the middle of the wrist (metacarpus). Make four columns on chart paper or the black board, labeled first, second, third, and fourth section. Have each student read off his or her finger measurements. Take the average for each column. The ratio of the second section to first section, the third to the second, and fourth to the third will all fall closely to tau.

Our navel also tends to divide our entire body into Golden Section, and our brows tend to divide our faces into the same ratio. The tau ratio can also be found in other growth patterns in nature. Old growth on many plants tend to be in Golden Proportion to new growth, etc.

As an extension activity students can be asked to build a structure of their own choosing, and the make copies of it in several Golden Section scales.

Assessment:

Study student's models, drawings, and notes in math journals. To meet the standard students must complete measurements of Zome System models and finger bones and derive ratios according to the activity in the lesson.

Standards Addressed:

- * Mathematics standards addressing investigation of mathematical connections (NCTM Standard 4).
- * Mathematics standards addressing the development of number and number relationships (NCTM Standard 5).
- * Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).
- * Mathematics standards addressing extensive concrete experiences using measurement (NCTM Standard 13).

Transfer Possibilities:

More work on number patterns and the occurrence of the Golden Section in nature and the man-made world ("What is Reflection Symmetry?," "Multiple Reflection Symmetry," "Fun Fibonacci," "The Golden Thread-A History of Tau," and "Fibonacci Numbers and the Golden Section"). Doing algebra and trigonometry with the Golden Section ("Similarity and the Golden Section," and pages 21-24 in "Zome System Manual").

Mathematics Intermediate Concept

Lesson Objective:

Students will determine the definitions of **polygons** and **polyhedra**, and identify several specific features of those shapes and solids. They will learn the Greek **nomenclature** for 2-D and 3-D shapes.

Prerequisite Skills:

Knowledge of basic polygons.

Time Needed:

One or two class periods of 45-60 minutes.

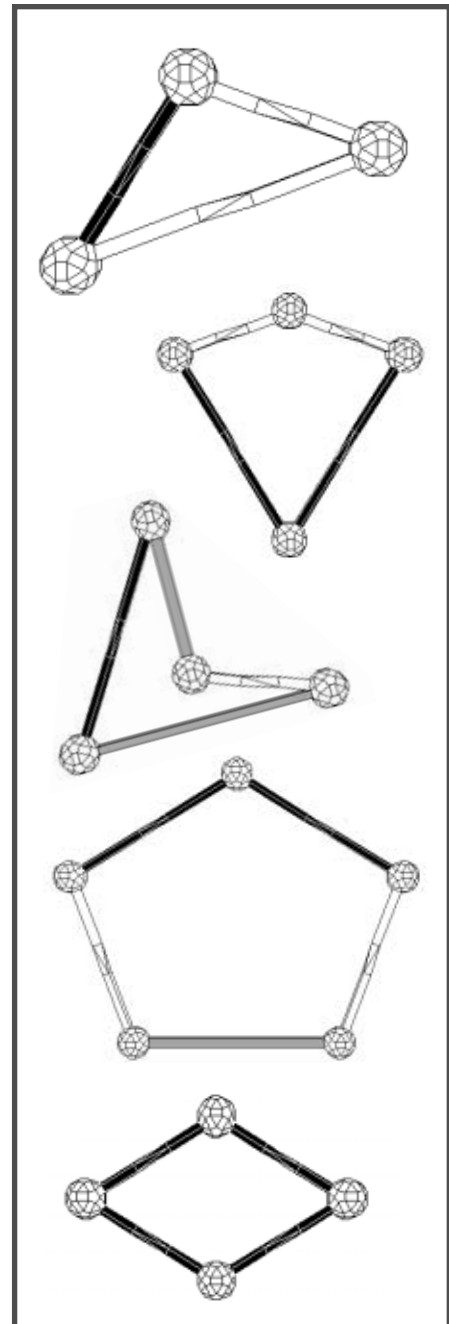
Materials Needed:

- One or two Zome System Creator Kits for 25-30 students

Procedure:

Divide the class into teams of 2-3 students, and distribute the Zome System elements evenly. Tell the students that they will investigate how to name 2-D and 3-D shapes. *Can anyone define a polygon? How many sides does a polygon have? Can they have curved sides? Can the lines cross? How many dimensions does a polygon have?* Ask the students to create a definition of a polygon in their math journals. For example; a polygon is a closed chain of points connected by straight lines in one plane. None of the lines may cross. The word comes from a Greek word which means “many angled.” A polygon is said to be **convex** if none of its vertices dent inward.

Have the students build some random polygons. *Do the polygons have to be flat? Allow the teams to discuss what they have made and settle on a system of sorting the polygons into groups. What is a good way to classify the shapes? By number of sides? By color? Do we put convex and concave polygons in separate groups?*



Naming 2-D and 3-D Shapes

Zome System

Builds Genius!

Now, have the teams build polygons from only blue struts. The polygons must have all equal sides and all equal angles. *What is the name of these special polygons? How many sides do they have?* These kinds of polygons are said to be regular polygons. Challenge the teams to build 3, 4, 5, 6, and 10-sided regular polygons.

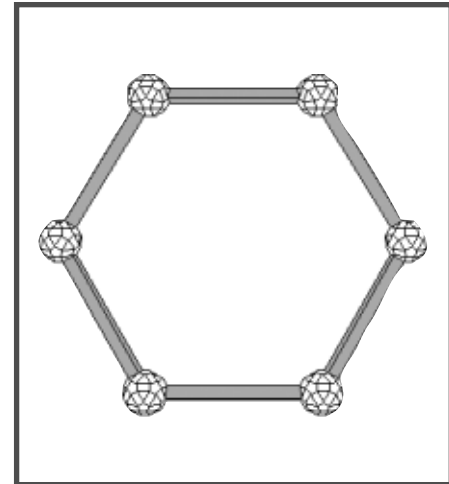
Who can name any of these shapes? What do the names tell us (how many angles they have)? Does anyone know where these names come from (derived from Greek words)? Why would we use Greek, of all languages? Why not English?

Allow the class to decide how they want to organize the Greek prefixes and numbers they represent. Encourage the students to make a connection between each Greek prefix and corresponding numbers. As each figure which the students built is named, have the students write the prefix and corresponding number into their math journal. Fill in the remaining numbers and prefixes as shown.

Using this list, the students should be able to name the polygons they built. Usually by convention, however, the higher number polygons can be named numerically. For example, a 37-sided polygon can be called a 37-gon.

Exceptions should be pointed out. For example, under the system just described, a 3-sided polygon should be called a triagon, instead of a triangle. However, in Greek, "gon" means angle, so it is still close to the same naming system. Also, a regular 4-sided polygon is called a square. Other 4-sided figures are called quadrilaterals, which is from the Latin for "four sides." *What is the real name for a square (regular tetragon)? For quadrilaterals (tetragon)?* Ask each group to present a few of their polygons to the class and explain how and who they gave it a certain name.

Once the class is comfortable naming polygons they should be challenged to apply the Greek naming system to 3-D shapes also. *We know that 2-D shapes are called polygons, but what are 3-D shapes called (polyhedra)?* A single 3-D shape is a polyhedron. A polyhedron is named by how many polygon sides or faces it has. For instance a four-faced polyhedron is called a tetrahedron. *What is the name of a polyhedron with 8 faces? With 12? What is the correct name of the shape we normally call a cube (hexahedron)?*



3	tria
4	tetra
5	penta
6	hexa
7	septa
8	octa
9	nona
10	deca
12	dodeca
20	icosa
30	triaconta

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Naming 2-D and 3-D Shapes

Depending on how much time you have available, allow the teams to build one or several polyhedra and apply the naming rules they have learned to them.

In general, what is the rule for naming 2-D and 3-D shapes? The students should create and write their own definitions in their math journal.

Conclude the class with a discussion on the advantages of conventions in naming geometric shapes. *Why is it useful for us to use the Greek names? Are there other subjects where we use terminology based on a foreign language? What is the purpose of standardization in general?*

Assessment:

Evaluate the polygons and polyhedra the students build and present. Study definitions and comments in math journals.

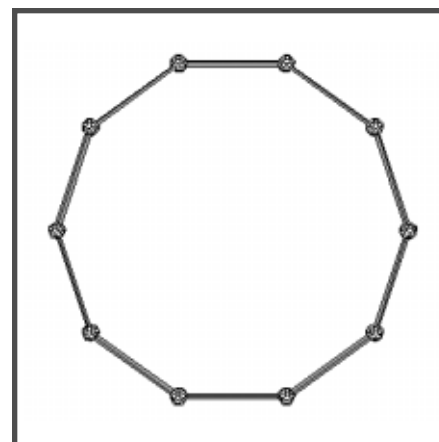
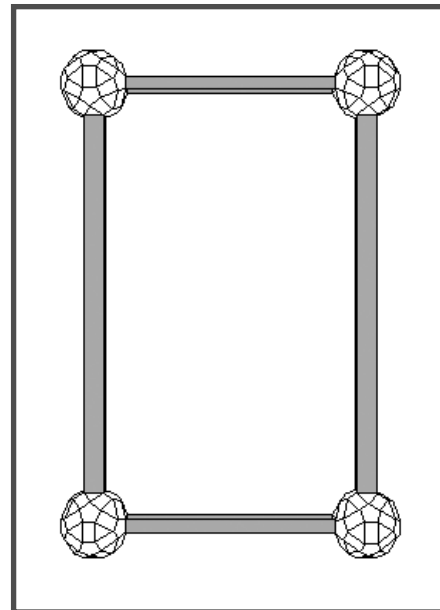
To meet the standard students must be able to build and name basic polygons and polyhedra. To exceed the standard they must verbalize a general rule for naming polygons and polyhedra.

Standards Addressed:

- * Mathematics standards addressing **mathematics as a means of communications** (NCTM Standard 2).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Study of 3-dimensional polyhedra (“Plato’s Solids - I,” “Plato’s Solids - II,” “Euler’s Formula for Polyhedra,” and “Archimedes’ Solids”). Discussions of the role of standardized language in math, science, and liberal arts.



Mathematics Intermediate Concept

Lesson Objective:

Students will start their exploration of regular polyhedra shapes. Based on how many polygon faces can meet at a vertex of a solid, they will determine that there only can exist five such 3-dimensional shapes.

Prerequisite Skills:

Knowledge of basic polygons ("Geometric Shapes"), and ability to define a two-dimensional versus a three-dimensional figure ("2-D and 3-D Shapes").

Time Needed:

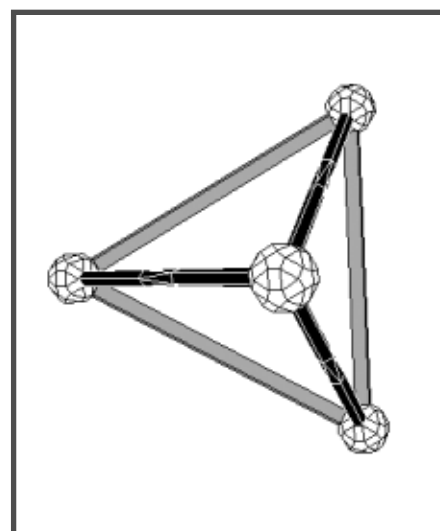
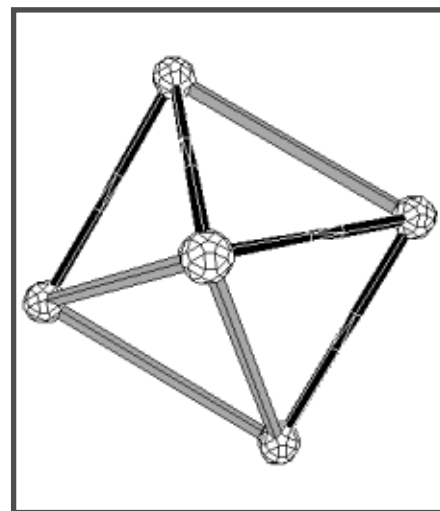
One or two class periods of 45 to 60 minutes.

Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Three or four packs of supplementary green Zome System struts, if available
- One pair of scissors per team
- One roll of tape per team
- Card stock polygons. See "Resource" section in this document.

Procedure:

Start the class with a brief review of polygons and polyhedra. *What is a polygon? What do their names tell us about them? What is a **regular polygon** (all vertices and angles are identical)? What is a **polyhedron**? Are polyhedra always made up of polygons? How are polyhedra named?* The present class will cover a special type of polyhedra called regular polyhedra. These are solids which have only regular polygons for faces and only one kind of face. The same number of faces meet in each vertex. Show a cube build with Zome System (blue struts) as an example of a regular solid consisting of square faces. All faces are squares, and three



Plato's Solids - I

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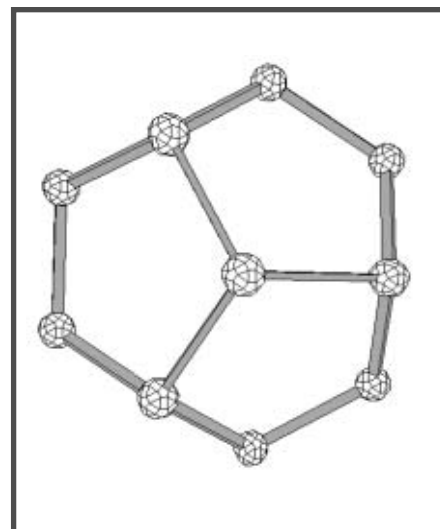
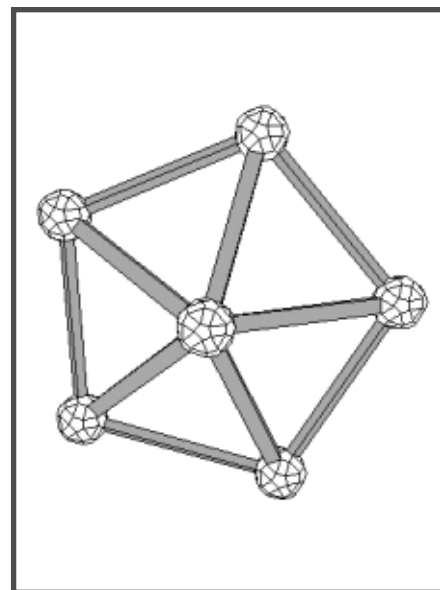
faces meet in each vertex.

Divide the class into teams and distribute the Zome System pieces, the scissors, tape, and card stock polygons. The task for the teams is to discover how many regular polyhedra there are. *How should we go about determining how many regular polyhedra exist? Is the number unlimited or are there a set number?* Ask the students to write down their predictions in their math journals. Discuss the various strategies that the students propose.

Let the students build for 20-30 minutes trying to create as many regular solids as possible. Discuss each solid the students offer as a solution. *Are all the faces the same polygon? Do the same number of faces meet in each vertex?* In this initial exploration it is acceptable to have solids where the faces are not regular polygons. How many did the students find?

Next students should focus on examining the vertices one at a time, using the various regular polygons, starting with the simplest. *What is the simplest regular polygon (the triangle)? Can there be different numbers of edges going into a vertex? What about the angles of the vertices? Can they vary? How many will make a vertex? Do two triangles make a full vertex? Do three?* The students should attempt to build each vertex in turn. If you have the supplementary green struts, you will be able to make all the vertices with regular triangles. If you do not, you must use a combination of red and blue struts to complete the vertex consisting of three triangles and the vertex consisting of four triangles. Tape the cardstock triangles together at the edges to reinforce how the triangles fit together. *What is the maximum number of triangle that will fit into a vertex (Five is the maximum. Six triangles will form a plane hexagon, thus not a vertex.)?*

When the limit is reached for triangles, move on to the next shape. *What is the next regular polygon? How many squares fit together to make a vertex? How many ways can squares make vertices? How many will make a flat fit?* Again, make models in Zome System of the vertices that work. *What is the next regular polygon? How many fit together into a vertex?* Continue this until the teams discover the limits.



Zome System

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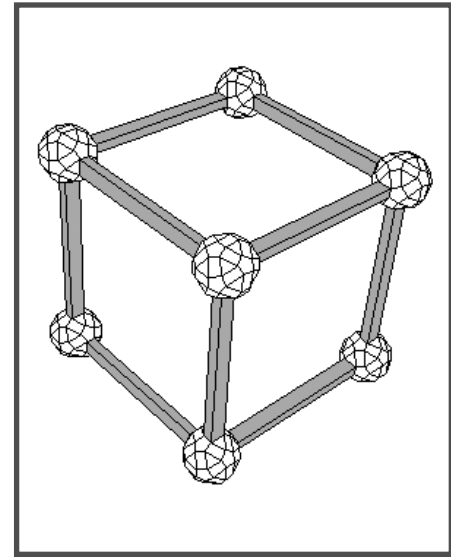
Plato's Solids - I

Hexagons fit together in a flat configuration only, and beyond six sides, the angles at the vertices are too big ($<120^\circ$) to put more than three together.

How many vertices have been found? Looking at just the vertices, can we tell what the completed solid will look like? Students should write down their observations and attempt to formulate a general rule describing the three-dimensional vertices that can be created using regular polygons. The teams should save the both the paper and the Zome System vertices they have built.

Regular solids have been studied all over the world for a very long time. The Greek philosopher Plato codified these forms around 2,400 years ago. He stated that only five solids could be created by repeating a regular two-dimensional polygon to complete a three-dimensional polyhedron. The criteria he used was that all features of the solids had to be regular, identical faces, angles, and vertices, without any overlap or gaps.

The class "Plato's Solids - II" continues the exploration of these polyhedra.



Assessment:

Observe and students as they build their structures, and take notes of their findings. Review notes in math journals.

To meet the standard, students must build both paper and Zome System vertices and determine how many are possible. To exceed the standard they must also formulate a rule which states why only five vertices are possible.

Standards Addressed:

- * Mathematics standards addressing **mathematical problem solving as a method of inquiry and application** (NCTM Standard 1).
- * Mathematics standards addressing **mathematics as reasoning** (NCTM Standard 3).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Continued exploration of Polyhedra ("Plato's Solids - II", "Archimedean Solids", and constructions 4, 5, 6, and 8 in Zome System Manual). More work on three-dimensional tessellations ("3-D Triangle Tiles" and "Beehive City").

Mathematics / Philosophy Intermediate Concept

Lesson Objective:

Students will build the five regular Platonic solids with Zome System. They will learn about the solids' history and their occurrence in nature, the man-made world, and philosophy.

Prerequisite Skills:

Knowledge of basic polygons ("Geometric Shapes"), and ability to define a two-dimensional versus a three-dimensional figure ("2-D and 3-D Shapes"). Completion of "Plato's Solids - 1".

Time Needed:

One class period of 45 to 60 minutes.

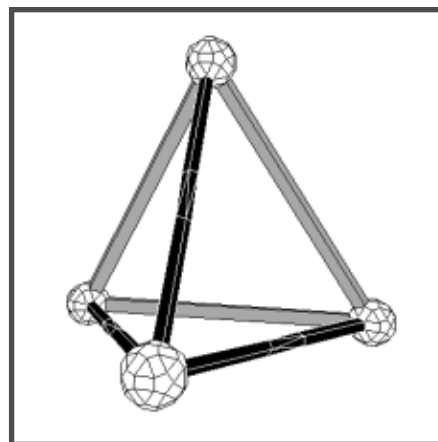
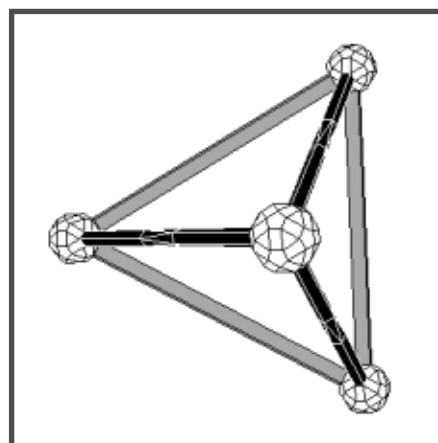
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Three or four packs of supplementary green Zome System struts, if available
- Cardstock and Zome System vertices built during the "Plato's Solids - 1" lesson
- Examples or images of crystals, virus and radiolarians, if available (can be found on the Internet)

Procedure:

Group the students into their teams from the previous lesson, and return to them the cardstock and Zome System vertices built then.

How should we go about building complete solids from the vertices we built before? How many solids can be made based on the five vertices? What would be the best way to proceed? Define for the students **regular polyhedra** if they are not familiar with the definition. Direct the students to build the complete solids, based on their suggestions of how to meet the challenge.



Plato's Solids - II

Zome System

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Students who do not have access to the supplementary green Zome System struts may encounter some problems building the tetrahedron or octahedron. A slightly irregular tetrahedron can be made using a blue equilateral triangle and built upward using red struts. A slightly irregular octahedron is made by creating a rectangle using blue and red struts, and then building upward and downward using red and blue struts. Both solids can be built regular using the green struts.

Categorize the shapes by the shapes of the faces they have, and name the shapes. *How do we name geometric shapes? How many faces do the five solids have?*

Name of Solid	Number of Faces	Shape of Each Face
Tetrahedron	4	triangle
Hexahedron (cube)	6	square
Octahedron	8	triangle
Dodecahedron	12	pentagon
Icosahedron	20	triangle

History of Plato's Solids

No one knows who first discovered these shapes, but excavations in Europe have unearthed carved dodecahedral toys, which are at least 2500 years old. Images of the solids were also made by civilizations in both Africa and South America. The first person to describe all of them together was the Greek philosopher Plato, about 400 BC. He believed that the solids have mystical properties associated with the four elements of alchemy: earth, air, fire, and water. Plato assigned each element to a solid.

Fire — Tetrahedron

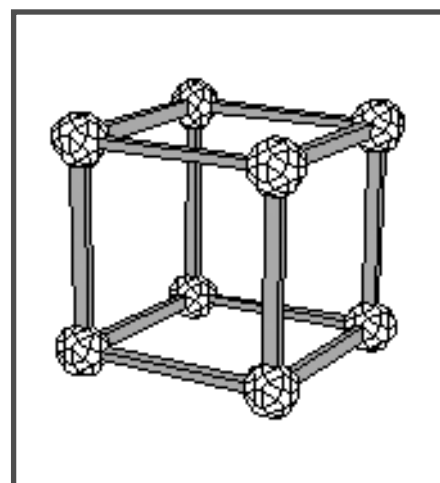
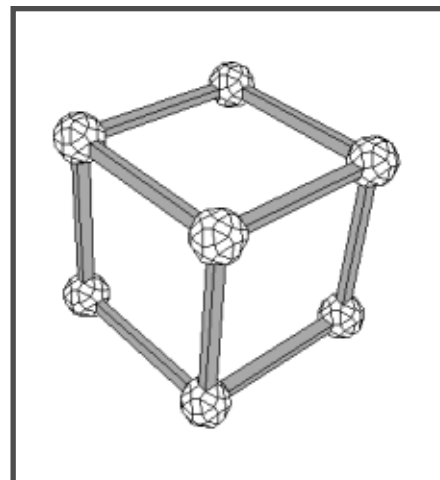
Earth — Cube

Air — Octahedron

Water — Icosahedron

This leaves the dodecahedron, which Plato associated with the shape of the whole universe, and the property of ether. *What do the solids have in common with their element?*

The 16th century German astronomer Johannes Kepler was fascinated by the Platonic solids, and came to believe



Zome System

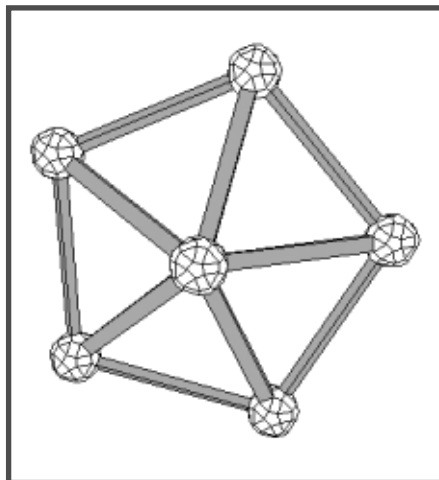
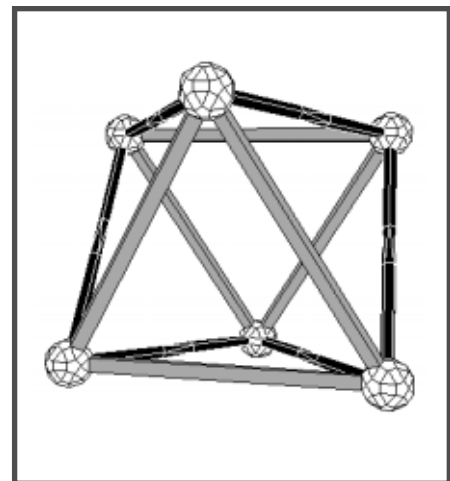
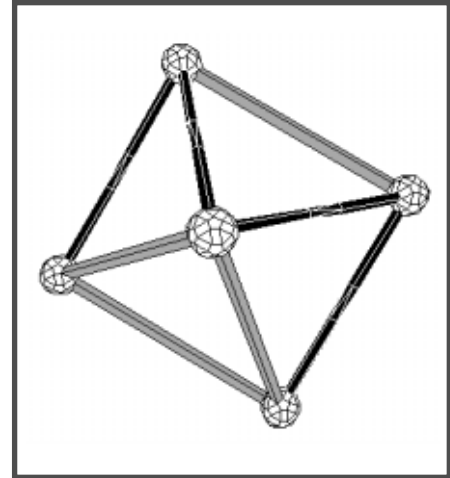
Builds Genius!

Plato's Solids - II

that they must be related to the structure of the universe. He created a model of the solar system based on the solids nested inside one another to explain the relative sizes of the planetary orbits. Kepler eventually abandoned this scheme as it never fit the real orbit sizes well enough. If time allows, the teams can attempt to nest the solids inside each other following Kepler's theory.

The five regular solids also occur in nature. For instance, many mineral crystals grow into the shape of an octahedron or a cube. Fluorite crystals exist in cubic and octahedron shapes. These are relatively inexpensive, and may be brought in for the class to examine. The external skeletons of microscopic sea creatures called radiolarian are shaped as tetrahedra. Most viruses, including those causing measles, AIDS, and the common cold are shaped as the icosahedron. The only molecule shaped like a dodecahedron, (Dodecahedrane $C_{20}H_{20}$) is made of carbon and hydrogen, the basic elements of life.

The solids continue to fascinate modern thinkers. The famous architect Buckminster Fuller based the design of his geodesic dome on the icosahedron. *Why does nature and man use such symmetrical shapes?* Ask the students to write down their theories of how the Platonic solids are connected to natural forms.



Plato's Solids - II

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Assessment:

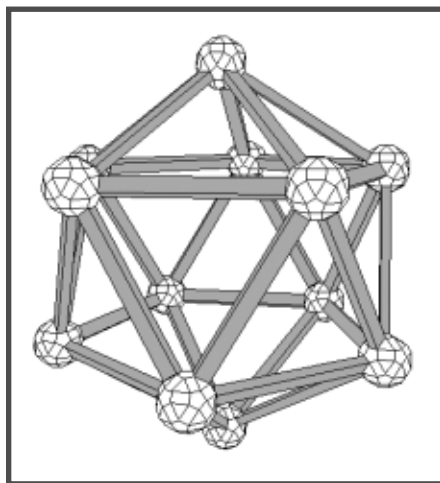
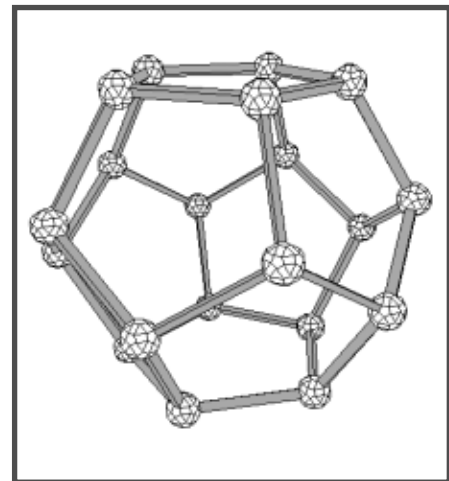
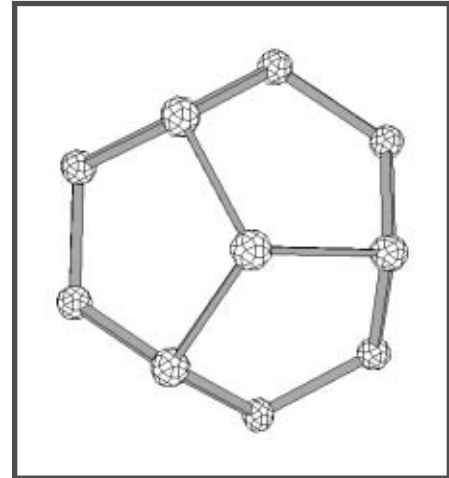
Observe and students as they build their structures, and take notes of their findings. Review notes in math journals. To meet the standard, students must complete their Platonic solids in Zome System. To exceed the standard they must make written connections between the solids and naturally occurring forms.

Standards Addressed:

- * Mathematics standards addressing **investigation of mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Continued exploration of Polyhedra shapes ("Archimedean Solids", and constructions 4, 5, 6, and 8 in Zome System Manual). More work on three-dimensional tessellations ("3-D Triangle Tiles" and "Beehive City").



Zome System

Builds Genius!

2-D and 3-D Stars

Mathematics Intermediate Concept

Lesson Objective:

Students will examine how the process called **stellation** can generate star shapes in two and three dimensions. They will also explore the concept of **geometric progression**.

Prerequisite Skills:

Knowledge of geometry nomenclature (“Naming 2-D and 3-D Shapes”). Ability to identify and build the icosahedron and the dodecahedron (“Plato’s Solids - I”, and “Plato’s Solids - II”).

Time Needed:

Two class periods of 45-60 minutes.

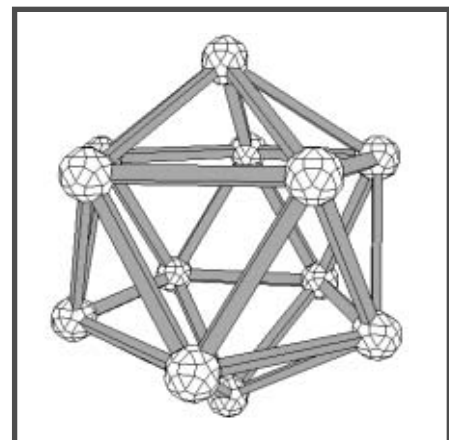
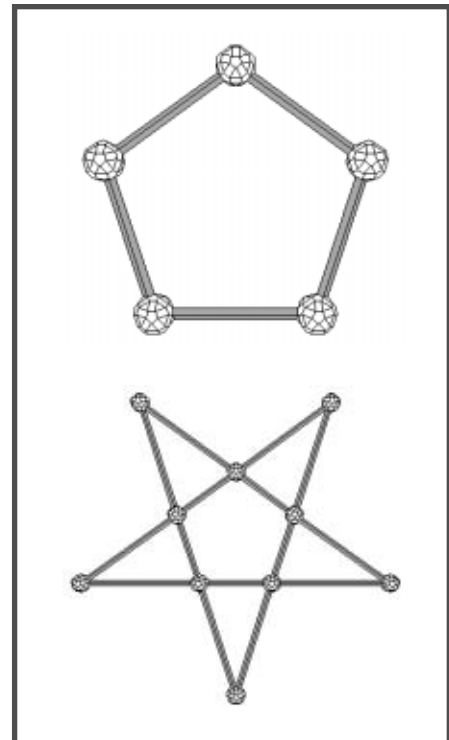
Materials Needed:

- Two Zome System Creator Kits for 25 - 30 students

Procedure:

Inform the class that they are going to explore an interesting process of changing geometrical shapes. Divide the class into teams of 3-4 students. The first challenge for the teams is to build an equilateral pentagon, and then extend the edges of the pentagon until these edges intersect with each other. *What does the resulting figure look like? How many arms does the shape have? Is it symmetrical?*

The process of extending the edges of a polygon or a polyhedron is called **stellation**. The resulting figure in this case is called an **edge stellated pentagon** or a **pentagram**. No one knows who first discovered this figure. We do know that the Greek mathematician and philosopher Pythagoras knew about it. His Pythagorean brotherhood used it as their secret symbol, known only to members. In fact, they drowned a fellow member for revealing it to a non-member.



2-D and 3-D Stars

Zome System

Builds Genius!

*What shape would we have if we connected the outer five points? Give the teams time to complete the figure. Which color strut did they have to use? Which length? Can the figure be stellated again? What do the outer points form? Are the pentagons increasing in size by the same amount every time? How long can this process continue? Increasing a shape the same way every time is called a geometric progression or a **fractal progression**.*

Can we stellate in this manner in three dimensions as well? Ask the students to build an icosahedron out of the shortest blue struts. Once the icosahedron is complete they should stellate it. Circulate and assist as needed as the teams find their own solutions to this challenge. Which edges should they be extending? Which struts will they need?

The stellation will yield a **star polyhedron**. *Based on the process of naming the 2-D star, what is a good name for the new*

3-D shape (edge-stellated icosahedron)? What kinds of shapes make up the points of the star (triangular pyramids)? How many such pyramids does the shape have (twenty)? What shape is defined by the outer points (dodecahedron)? Can this resulting polyhedron be stellated in turn? What is the name of the resulting star polyhedron (edge-stellated dodecahedron)? What shape are the points (pentagonal pyramids)? How many such pyramids does the shape have (twelve)? What shape is defined by the outer points now (icosahedron)? Does this sound familiar? Can we continue this to make it larger? How about smaller? What kind of progression is this geometric or fractal?

The students should complete the lesson by writing a definition of stellation in their math journal. They should also attempt to describe the duality of the icosahedron and the dodecahedron.

Assessment:

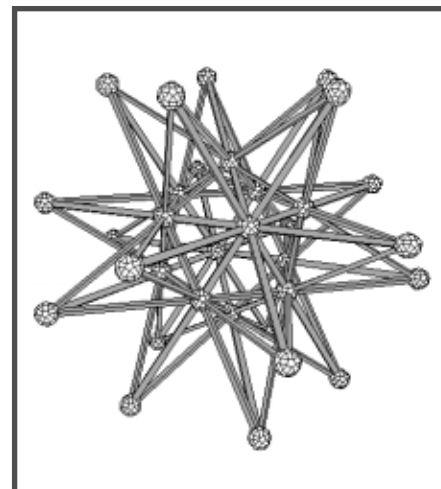
Observe and students as they build their structures, and take notes of their presentations. Review definitions in math journals. To meet the standard, students must build the 2-D and 3-D stellations. To exceed the standard, their definitions must convey the geometric progression of the stellations.

Standards Addressed:

- * Mathematics standards addressing **mathematics as a means of communications** (NCTM Standard 2).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Other transformations of geometrical shapes ("Archimedean Solids"). Also explorations of geometric growth patterns in the natural world ("Multiple Reflection Symmetry" and "Fun Fibonacci").



Mathematics / Art Intermediate Concept

Lesson Objective:

Students will learn that while regular pentagons can not tile a plane by themselves, they can tile with a few other shapes. Students will identify these tilings as being **non-periodic**.

Prerequisite Skills:

Some previous work with tessellation ("Trying Tessellation," "What are Quadrilaterals" "Tiling with Quadrilaterals," and "Plane Patterns"). Basic understanding of symmetry concepts ("What is Reflection Symmetry?" "Multiple Reflection Symmetry," "Rotational Symmetry," and "Translational Symmetries in Tilings").

Time Needed:

Two class periods of 45-60 minutes.

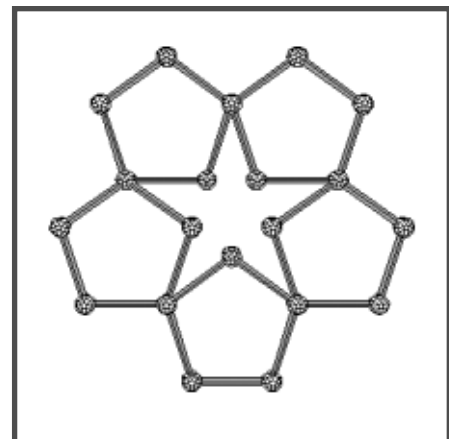
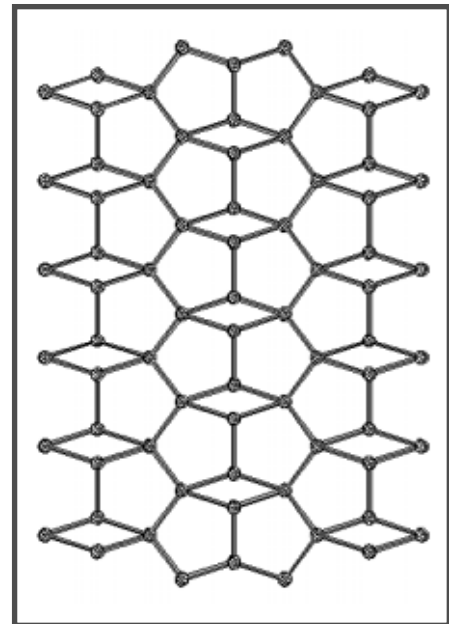
Materials Needed:

- Two Zome System Creator Kits for 25-30 students

Procedure:

Review with the students the tilings created in the previous lesson. *What is special about the repetition in those tilings? How many different shapes were used in one tiling?* When one or several shapes are repeated in a continuous pattern, the tiling is called **periodic**. When the pattern changes or is interrupted, the tiling is called **non-periodic**. People around the world have worked with tilings and mosaics of many kinds. The German scientist and astronomer **Johannes Kepler** (1571-1630), who explored non-periodic tilings extensively. The tilings in this lesson are often known as **Kepler Tilings**.

Divide the students into teams, and distribute the Zome System pieces. The teams should build simple periodic



Kepler's Tilings

tilings with regular polygons, and of course, no gaps between the shapes. For triangles, squares, and hexagons, we see clearly that there are no gaps between shapes, and that a fixed number of the tiles fit evenly around a point with no gaps.

What about pentagons? Can we tile them with no gaps? What would be a good way to find out? Have the students build a regular pentagon, and attach more pentagons around the vertex. How many pentagons can we fit around the vertex? Does it fit perfectly? Does this mean we cannot tile with them? To discover what happens, have the teams continue building with pentagons. How many ways can we tile with pentagons plus one other shape? Which other shapes are needed to complete the tiling? Consider different ways to form a ring of pentagons with a gap in the middle. Several examples are shown in the figures.

In the different tilings of the teams, which ones have transitional symmetry? Which have rotational symmetry? Do any teams have tilings with no symmetry at all?

Conclude by having the students write their own definitions of non-periodic tilings in their math journals.

Assessment:

Study the tilings produced by the teams and review definitions in math journals. To meet the standard students must determine which shapes can be used to tile with regular pentagons. To exceed the standard their definitions must clearly distinguish between periodic and non-periodic tilings.

Standards Addressed:

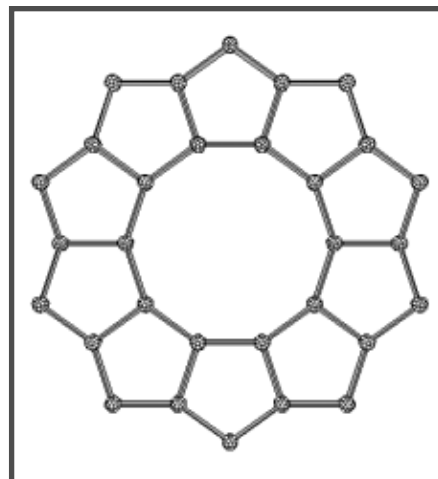
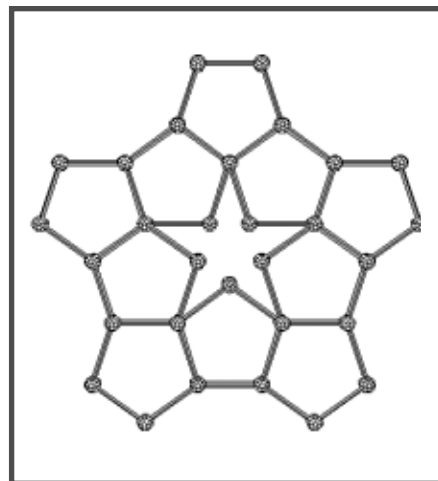
* Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

More advanced tilings in 2, and 3 dimensions ("Richert - Penrose Tilings," "3-D Triangles," "3-D Triangle Tiles," "Plato's Solids - I," and Plato's Solids - II".)

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Richert-Penrose Tilings

Mathematics Intermediate Concept

Lesson Objective:

Students will discover Richert-Penrose tiles which can tile the plane non-periodically, without translational symmetry.

Prerequisite Skills:

Some previous work with tessellation (“Trying Tessellation,” “What are Quadrilaterals” “Tiling with Quadrilaterals,” and “Plane Patterns”). Basic understanding of symmetry concepts (“What is Reflection Symmetry?” “Multiple Reflection Symmetry,” “Rotational Symmetry.” and “Translational Symmetries in Tilings”). Knowledge of non-periodic tilings (“Kepler’s Tilings”).

Time Needed:

One class period of 45-60 minutes.

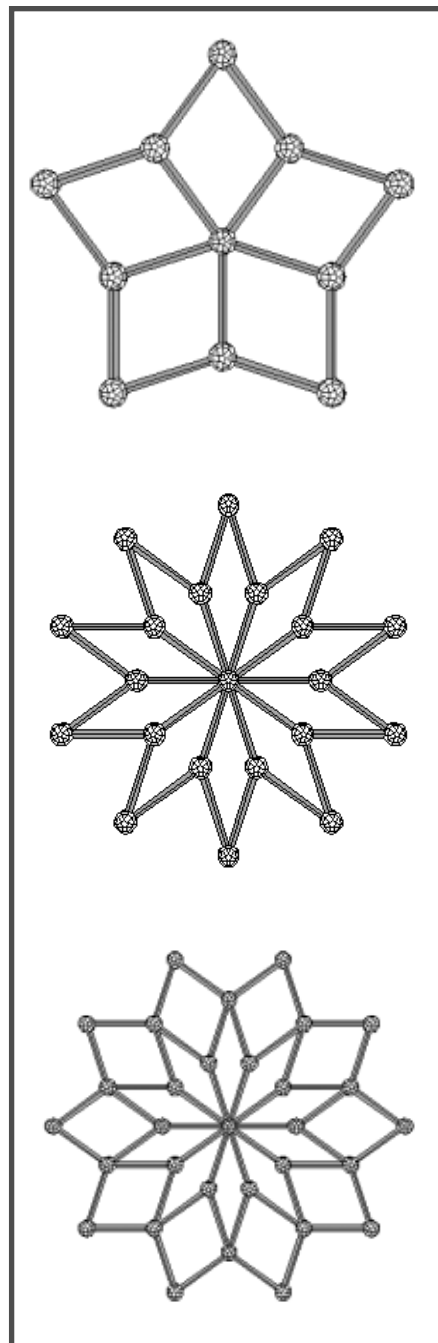
Materials Needed:

- Two Zome System Creator Kits for 25-30 students

Procedure:

Divide the class into teams of 3-4 students, and distribute the Zome System pieces evenly. Each team should start by making a flat starburst of 5 blue struts of one length. This will form a starfish with arms 72° apart. *Can we make these V shapes into diamonds? How can we continue building outward in this manner? What would the pattern look like? Can we continue farther? What shapes do we have? Are they identical? What are the angles on each? What kind of symmetry does this pattern have? Is there a way to determine the angles without measuring?*

Next, have each team begin by filling in all ten holes on the equator of a node with the pentagon shape on top. *Can all of the Vs be made into diamonds? Can this be continued?*



Richert-Penrose Tilings

How far? How many shapes were needed? What kind of symmetry does this pattern have? Does this pattern have more than one kind of vertex? How many? How many distinctly different vertices can be created with these two diamonds. i.e. how many combinations of skinny and fat diamonds exist around a central node?

A typical random vertex is shown in the figure. *Can a tiling be made with no symmetry at all, completely random?* Conclude by exploring how many can be identified with the remaining time.

The properties of the tilings in this lesson were discovered independently by the American artist Clark Richert and by the British mathematician Roger Penrose in the late 1960s and early 1970s.

Assessment:

Study the tilings produced by the teams and review definitions in math journals. To meet the standard students must determine how many ways the skinny and fat diamond can combine around a central node. To exceed the standard they must be able to state whether a Richert-Penrose tiling is always non-periodic, or if it can be made periodic.

Standards Addressed:

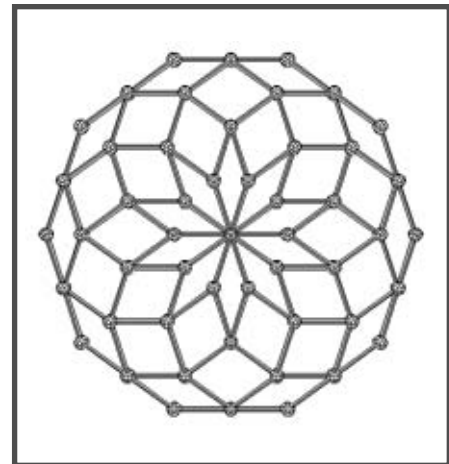
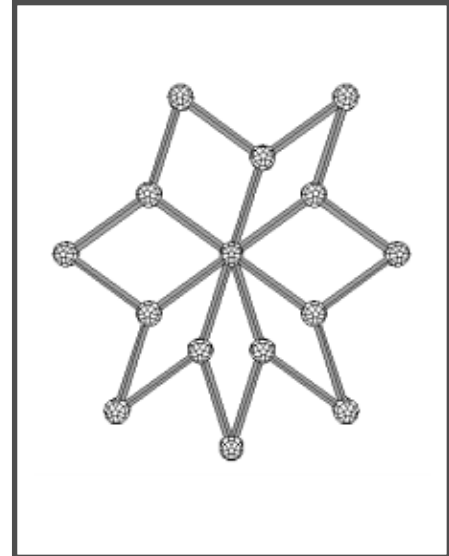
* Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

More advanced tilings in 3 dimensions (“3-D Triangles,” “3-D Triangle Tiles,” “Plato’s Solids - I,” and Plato’s Solids - II”).

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Bridge Building Project

Integrated Unit Intermediate Concept

Unit Objective:

In this unit students will design and build models of a bridge intended for a local creek or river. They will learn how to develop plans and descriptions of the bridge, calculate a budget covering all project costs, and build scale models with Zome System elements. They will also make written and oral presentations of their designs. During the project students will address numerous concepts including measurement (metric and British/US standard,) basic geometric concepts such as lines, rays, angles; shapes, perimeter, area, scale, and decimals, budgeting and accounting, technical writing, presentation skills, and science concepts including stress and gravity.

Prerequisite Skills:

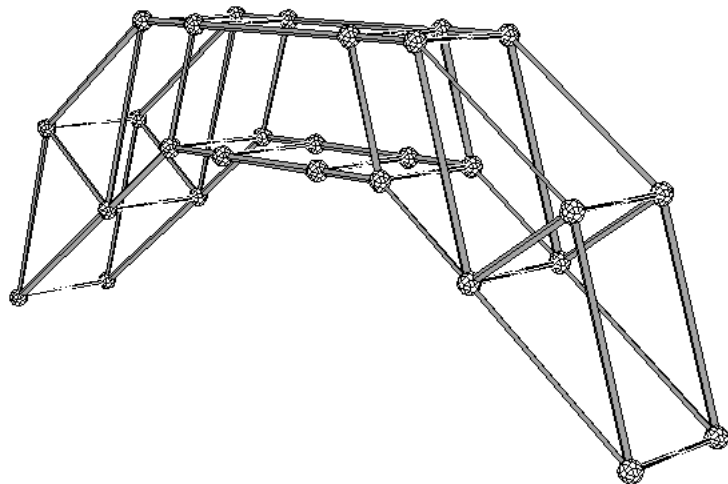
Students must know basic geometric shapes (“Geometric Shapes”). They should also be able to use a protractor (Attention Angles!,” and “Trying the Triangle”).

Time Needed:

The unit may take between 6 and 9 weeks if your students work on it for 45 minutes per day. The exact time will depend on how deeply you concentrate on each component.

Materials Needed:

- Two Zome System Creator Kits for 20-24 students
- Building project plans and blueprints of different types: bridge plans, house plans etc. Can be obtained at the library, through architects and building companies, and at some local hardware stores. Some schools may have plans made by students in earlier years.
- any good books on bridges, buildings, and structures
- protractors
- cardboard
- meter stick, yard stick, rulers
- three old telephone books (metric and British/US standard)
- masking tape
- graph paper
- tape measure



Bridge Building Project

Zome System

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Procedure:

The unit contains 20 components, each teaching different concepts. The four components listed first use Zome System, and are described in detail. The later 16 components are outlined briefly below, and should be planned using other resources. In many cases it is appropriate to run several component simultaneous. Each group of students can work on a component until they have completed all its tasks. As your students complete the self-directed assignments they can rotate to the next station.

graphic of bridge built with Zome System

Explain to your class that they are about to start a project that will take them several months to complete. They will be divided into teams of 3-4 students for the duration of the project. Each team will form a small construction company that is competing with other companies to build a bridge over a local creek or river. The County Commission (substitute applicable governmental entity) will choose which company will win the bid. The bidding process will include the tasks listed below. You may want to post the task descriptions in a prominent location in your class room.

- giving the Commissioners plans and descriptions of the bridge
- presenting the Commissioners with a Zome System model of the bridge that follows the building code.
- presenting an itemized budget of all costs, including construction materials, salaries, and land.

1. Metric Measurement. A math station intended to prepare the students for the bridge building unit. Students should read and follow the directions on the attached instruction sheet. The instruction sheet, which should be photocopied for each student, also serves as an evaluation form for the teacher.

2. Making Plans. A math station to prepare the students for the bridge building unit. Students should read and follow the directions on the attached instruction sheet. The instruction sheet, which should be photocopied for each student, also serves as an evaluation form for the teacher.

3. Construction Crew Wanted. A math and construction station providing a shorter practice project to prepare the students for the bridge building unit. Students should read and follow the directions on the attached instruction sheet. The separate evaluation sheet should be photocopied for each student.

4. Building to Scale. A math station providing the students with a greater knowledge of the concept of scale. Students should read and follow the directions on the attached instruction sheet. The instruction sheet, which should be photocopied for each student, also serves as an evaluation form for the teacher.

5. Create a Tile. This component is needed to review the basic concepts of symmetry and shapes.

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Bridge Building Project

Zome System lesson plans on tiling and tessellation are recommended (“Trying Tessellation,” Plane Patterns,” “Triangle Tiles,” and “Translational Symmetries in Tilings”).

6. Angles. You need to provide one component addressing the measurement of angles. Students need to know how to differentiate between and build right, obtuse, and acute angles. Zome System lesson plans on angles are recommended (“Attention Angles!”).

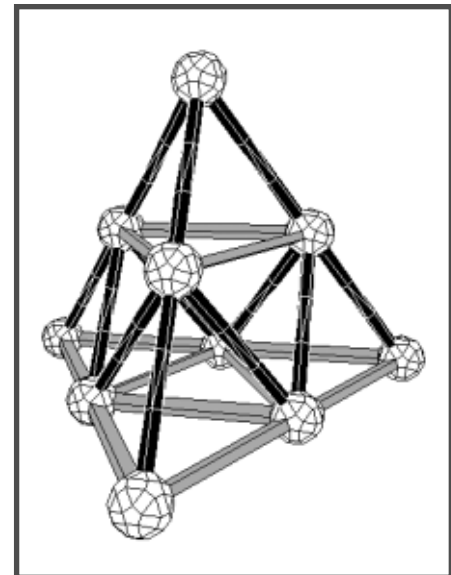
7. Mighty Triangle. One station should be devoted to experiments with weight and rigidity. One goal for these directed experiments should be to show the students that they need to incorporate triangular trusses in their bridges. The trusses can of course be built with Zome System (3-D Triangle Tiles).

8. Feel Like a Building. Students need a general section dealing with different architectural features. You should expose them to both engineering elements such as trusses, and to design consideration and architectural styles. Most libraries have resources to address this component.

9. History of Bridges. This component should focus on the historical development of bridge design and construction. Your students can gain this historical perspective through books, videos, and museum visits. Their research will help them make to connections to history, culture, and art.

10. Bridge Design. A more specific component examining different types of bridge design and their purposes. Use Zome System to build the various bridge features.

11. Soil Samples. Studies of soil and geology will help your students adapt their bridge design to suit their chosen location. This learning can be achieved with site visits, lab work and text books.



Bridge Building Project

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12. Hydrology. Theoretical hydrology is important when deciding how a bridge may be affected by erosion of the river's banks and its bottom. There are several good books and videos in the market that deal with this topic.

13. Build a River. A lab component demonstrating water flow and erosion. Many children's museums have displays of this type, and a classroom version can be made with a trough, a garden hose, and sand and gravel.

14. Field Trip. A visit to the creek will prompt discussion of the environmental effects of construction projects. Your students can also document terrain features that they will have to take into account when designing their bridge.

15. Newspaper Articles. Ask your students to read about high school model bridge building contests and related topics.

16. Speakers. Attempt to bring in parents or other people from the community who work in professions that can be related to the bridge unit. Examples include civil engineers (builds structures), geologists (soil samples), construction foremen (working from building plans), naturalists (environmental impacts), etc.

17. School to Work. It is helpful to use one section for a discussion of occupations and skills that are required in the type of construction projects the students are modeling.

18. Budgets. Students will need a comprehensive session on the concept of projected cost in order to make their projects realistic. In addition to traditional text books, it is possible to obtain materials on budgeting and economics from various business organizations.

19. Reflective Essay. Student essays should contain rationale for their site selection, bridge design, and other concepts they have learned from the unit.

20. Project Presentation. In this final component the students will make an oral presentation of their proposed bridge to a panel of "commissioners" (can be parents, teachers, or other students). The presentation should address the selected site, design of the bridge, and their budget. Other features that can be required include a display of detailed building plan on graph paper, and graphic representation of the budget.

Assessment:

Assessment will involve continuous student observation, individual and group questioning, and review of their presentations, essays, and the notes taken during each component. Achievements required to meet and exceed the standards will vary with each component.

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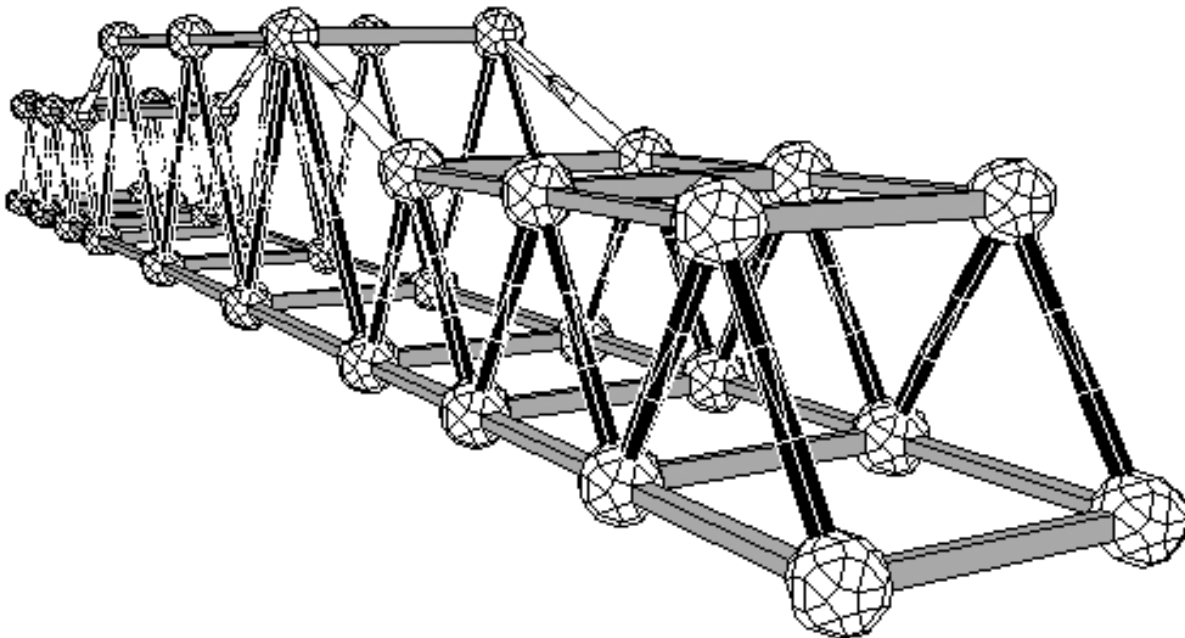
Bridge Building Project

Standards Addressed:

- * Mathematics standards addressing **mathematical problem solving as a method of inquiry and application** (NCTM Standard 1).
- * Mathematics standards addressing **investigation of mathematical connections** (NCTM Standard 4).
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).
- * Mathematics standards addressing **extensive concrete experiences using measurement** (NCTM Standard 13).
- * Language Arts standards requiring students to **write and speak for a variety of purposes**.
- * Language Arts standards requiring students to **apply thinking skills to their reading, writing, speaking, listening, and viewing**.
- * Physical Science standards requiring students to **know and understand common properties, forms, and changes in matter and energy**.

Transfer Possibilities:

The unit provides numerous connections to other subjects. Examples include architectural design and other forms of applied arts, city planning policy ("The Livable City"), more advanced work on budgeting and business finance, etc.



Making Plans

Materials needed: 10-20 of each Zome System element. Protractors, rulers, graph paper. Building plans provided by your teacher.

Name: _____ Date: _____

Step I

Select one set of plans. Measure the angles using a protractor and write them down. Did all members of your group get the same results?

Step II

Build a simple three-dimensional Zome System structure.

Step III

Make plans of your structure on graph paper. You must include the angle dimensions. Remember you must have your name on the plans.

Step IV Bonus

Make your plans in more than one color.

Metric Measurement

Materials needed: math textbook or dictionary, meter stick, yard stick, rulers (standard and metric), 10 of each Zome System strut.

Name: _____ Date: _____

Step I

Look up the word metric in your math book, write the definition:

Look up the word meter, write the definition:

Look up the word centimeter, write the definition:

Examine the meter stick. How many centimeters are there in a meter?

Which is longer the yard stick or the meter stick?

Why?

Measure the Zome System struts in centimeters (cm) then in inches (in.).

	<u>Centimeters</u> (cm)	<u>Inches</u> (in.)
1. short red		
2. short blue		
3. short yellow		
4. medium red		
5. medium blue		
6. medium yellow		
	<u>Centimeters</u> (cm)	<u>Inches</u> (in.)
7. long red		
8. long blue		

9. long yellow

How are centimeters and inches similar to each other?

How are they different?

Look at your ruler again, the smallest unit of measurement on the centimeter side is a millimeter (mm). How many millimeters in a centimeter? In a meter? Measure the Zome System struts in millimeters and place this column next to the inches column above. Which measurement system do you like best? Why?

Step II

Construct three two-dimensional (flat) Zome System shapes. Using your measurement chart add up the centimeter measurement of each strut for each shape. Draw your shape and then write the totals below.

Shape 1

Shape 2

Shape 3

_____cm

_____cm

_____cm

Look up the word perimeter in your math book, write the definition:

What would be the perimeter of each shape in inches?

Shape 1 _____ in. Shape 2 _____ in. Shape 3 _____ in.

Construction Crew Wanted

Materials needed: half set of Zome System, three old phone books

Each crew will:

- create the tallest possible structure that will support three phone books
- stay within a budget of \$2,400,000
- keep complete, accurate records of all your experiments
- complete the evaluation form

Each crew must purchase their supplies from Zome System Building Supplies.

<u>Building materials</u>	<u>Cost</u>
Nodes	\$1,000 each
short:	
red struts	\$500 each
blue struts	\$525 each
yellow struts	\$550 each
medium:	
red struts	\$1,500 each
blue struts	\$2,000 each
yellow struts	\$2,500 each
long:	
red struts	\$3,000 each
blue struts	\$3,500 each
yellow struts	\$4,000 each

Evaluation Sheet for Construction Crew Wanted

(to be done individually)

Name: _____ Date: _____

1. What did you learn about structures when doing this station?
2. What did you learn about budgets by doing this station?
3. How is this budget the similar to those you have done in other classes.
4. How is this budget different than those you have done in other classes?
5. If you could repeat this task what would you do differently?
6. If you could repeat this task what would you do the same?
7. On a scale of 1- 5, 5 being excellent, how did your group work together?
8. Why did you give your group that rating?

Building to Scale

Materials needed: 10-20 of each of the blue struts, 40-60 nodes, making tape, tape measure

Name: _____ Date: _____

Step 1

Measure all of the blue struts using the inches side of your ruler.

Short blue strut _____ in. Medium blue strut _____ in.

Long blue strut _____ in.

Step 2

Convert the inches to feet. The scale is 1" = 1 ft.

Short blue strut _____ ft. Medium blue strut _____ ft.

Long blue strut _____ ft.

Step 3

Using a tape measure, measure the scale footage. Take masking tape and mark the scale footage on the floor. Lay the Zome System on the floor next to the tape.

What do you observe?

What's the same in both?

What's different?

Step IV Bonus

Build a simple structure using the blue struts. Draw your structure on graph paper labeling the measurements in both inches and feet.

Zome System Warehouse Price list

Materials

Land (cardboard)	\$500,000
Building Materials (all colors)	
Short struts	\$10,000 per 5
Medium struts	\$15,000 per 3
Long struts	\$25,000 per 3
Welding material (nodes)	\$250 each
Building plan paper (4 sheets graph paper)	\$25,000
Extra sheets of graph paper	\$10,000 per sheet

Services:

Audit Service	\$2,000
Conflict Management	\$5,000

Fines

Your company may be fined for any of the following infractions:

• disturbing other companies	\$400
• leaving messy construction sites	\$550
• handling bridges that are under construction by other companies	\$1,000

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Euler's Formula for Polyhedra

Mathematics Intermediate Concept

Lesson Objective:

Students will discover Euler's formula for polyhedra, and will be able to show that it works for any convex polyhedron.

Prerequisite Skills:

Ability to build and identify different polyhedra including the 5 platonic solids ("Naming 2D and 3D Shapes," "Plato's Solids - I," and "Plato's Solids - II").

Time Needed:

One or two class periods of 45-60 minutes.

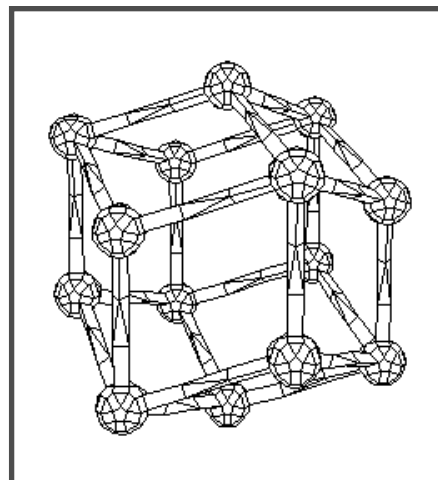
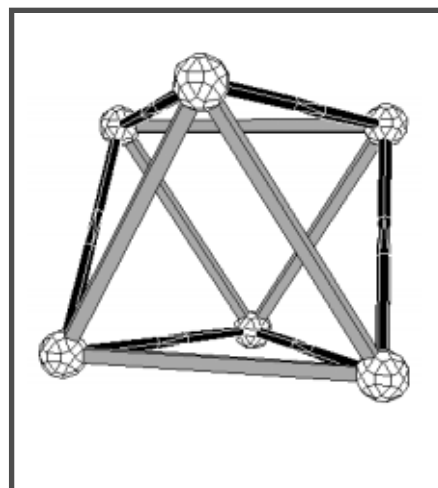
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- A potato or a piece of Styrofoam
- Sharp knife

Procedure:

Start the lesson with a brief review of what your students know about polyhedra. *What is a polyhedron or a solid? How are they named? Can anyone name the Platonic solids? How many are there? Who can name any other solids?*

List on the board the solids from the table below. Propose to the students that there is a numerical relationship between the faces, edges, and vertices of any given polyhedron. *How could we go about finding this relationship?* Divide the class into teams of three to five students. Their challenge is to find the general relationship formula. The teams should start by building one of each of the models from the list on the board. They should copy the names of all the listed solids into their math journals and create a table for the number of faces, edges, and vertices, using the shapes they have built to count each feature.



Euler's Formula for Polyhedra

Zome System

Builds Genius!

When completed their tables will look as follows:

	Faces	Edges	Vertices
Tetrahedron	4	6	4
Octahedron	8	12	6
Hexahedron (Cube)	6	12	8
Icosahedron	20	30	12
Dodecahedron	12	30	20
Triangular Prism	5	9	6
Pentagonal Prism	7	15	10
Pentagonal Pyramid	6	10	6

The students should continue their discussions until they find a pattern in these numbers in the form of a relationship between the edges, faces, and vertices of each shape. Try adding and subtracting the numbers in various combinations until they find a formula which gives the same answer in every case.

Once they derive the correct formula, write it on the board:

$$\text{Faces} + \text{Corners} = \text{Edges} + 2$$

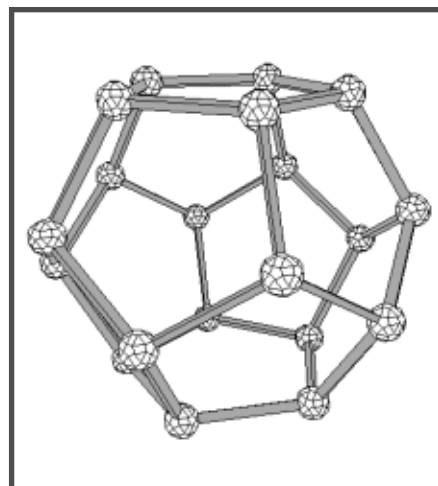
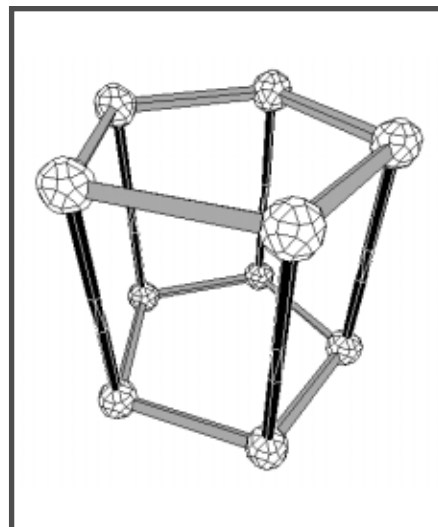
$$F + C = E + 2$$

or, $F + C - E = 2$

This formula is named "Euler's Formula" (pronounced "oiler") after the Swiss mathematician Leonhard Euler who discovered this relationship in 1752. Euler showed that his formula works for any convex polyhedron, whether it is regular or irregular.

If time allows, perform a demonstration of this using a potato (or a piece of Styrofoam) and a sharp knife. Cut slices from the potato until a random flat-faced polyhedra is produced. Count the faces, edges, and vertices, using a marker to keep track of the count. Demonstrate on the board that the totals fit into Euler's Formula. Alternatively, you could let the students build a number of irregular polyhedra and check that the formula holds up.

Can anyone think of a way this formula might be useful? For example, if a builder knows the number of bars and connectors in a dome, he or she can predict the number of



Zome System

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Euler's Formula for Polyhedra

panels required to build the dome.

Assessment:

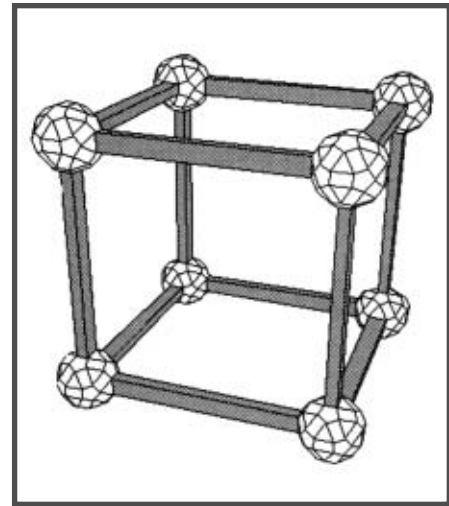
Review tables and formulas in student math journals. To meet the standard students must build and name the listed polyhedra, count their components, and attempt to construct a general formula. To exceed the standard they must correctly verbalize Euler's formula.

Standards Addressed:

- * Mathematics standards addressing mathematical problem solving as a method of inquiry and application (NCTM Standard 1).
- * Mathematics standards addressing mathematics as a means of communications (NCTM Standard 2).
- * Mathematics standards addressing mathematics as reasoning (NCTM Standard 3).
- * Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Further exploration of polyhedra geometry ("Archimedean Solids," and constructions 4, 5, 6, and 8 in Zome System Manual). Discussions of constructing proofs for mathematical formulas.



Mathematics / Biology Intermediate Concept

Lesson Objective:

Students will learn about Fibonacci numbers by discovering them in plant symmetries.

Prerequisite Skills:

Experience with symmetry lines in geometry and natural objects. (“What is Reflection Symmetry,” “Multiple Reflection Symmetry,” and “Rotational Symmetry”).

Time Needed:

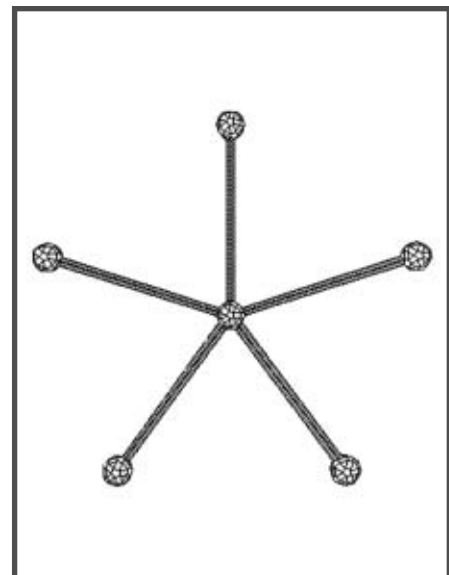
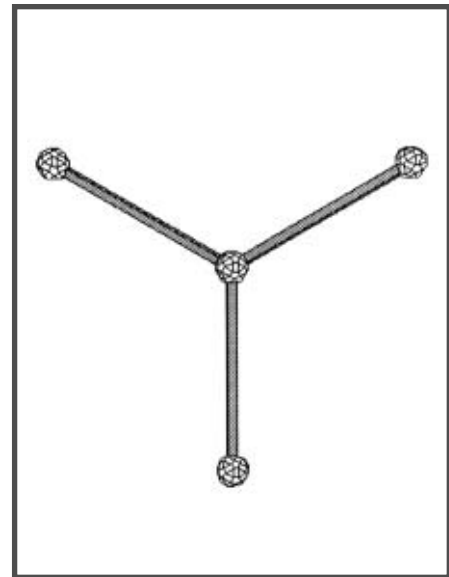
One class period of 45 to 60 minutes

Materials Needed:

- One Zome System Creator Kit
- Several pine cones
- One Pineapple
- One head of cauliflower
- One artichoke
- Sunflower-several different types and sizes
- Poster or other display of Fibonacci numbers in nature (see Resource section)

Procedure:

Divide class into teams of 3-4 students, and distribute the Zome System pieces. Have short review session on symmetry, Zome System geometry, and numbers in nature. *What are the three shapes shown on the Zome System node? What numbers do these shapes represent? Where do we find the numbers 2, 3, and 5 in nature? What symmetries do we find in plants?* If the class does not list them, some examples are 3-fold symmetry in green peppers, clovers, irises, bananas; 5-fold symmetry in apples, morning-glories and many types of flowers, and 2-fold symmetry in almonds and walnuts. Ask the students to build some simple polygons and show how many lines of symmetry they have.



Fun Fibonacci

Zome System

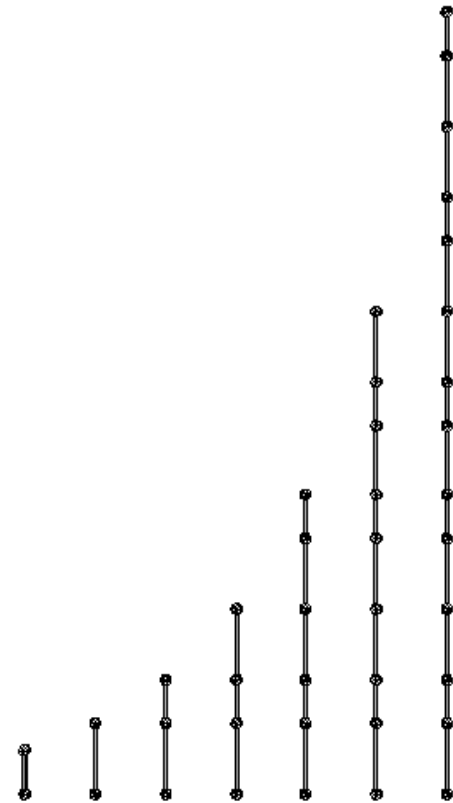
Builds Genius!

The question for the present class is if there are other numbers that can be found in natural objects. Distribute the fruit and vegetable examples among the students and allow them a couple minutes to try to determine where the numbers might be occurring. *If we begin looking at spirals in the plants, what might we see? Are the spirals clockwise or counterclockwise? How many clockwise spirals are on your particular botanical sample? Have the teams make a chart on of type of sample, number of spirals clockwise, and number of spirals counter-clockwise. How many counter-clockwise spirals are there? Use caution not to double-count the first spiral. Use a pins or a marker to keep track of the starting point. Are the spirals equal in both directions? Is one spiral tighter than the other? Have the students list the number of spirals they found, in order. What pattern emerges? Give students time to discover the additive nature of the series.*

Write the numbers 2, 3, 5 on the board. *How do these numbers relate to each other? How can they be used to make each other ($2+3=5$)? What is the next number in the sequence ($3+5=8$)? Continue questioning until the class has derived the entire sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233... How do these numbers in the series compare to the numbers you found in the spirals of the botanical samples? Are there similarities?*

This series, known as the **Fibonacci Sequence**, was discovered in the late 12th century by the Italian mathematician Leonardo of Pisa, also known as Fibonacci. Although he was the first person to write down this series of numbers, he had no idea that these numbers occurred in plants.

If we take a look at the three lengths of one color in Zome System, we might see that this series appears here as well. *How do the three lengths of one color resemble the Fibonacci Series?* There are two connections. First, the short and medium struts joined together are exactly the same length as the long one. Second, each strut is larger than



the one before it by the same multiple.

Conclude the lesson by displaying additional manifestations of Fibonacci numbers in nature. Posters and various activity books can be used.

Assessment:

Review the tables created by the teams, as well as notes in individual math journals. To meet the standards, students must be able to show symmetry numbers in the botanical samples. To exceed the standards they must draw connections between the numbers in their tables and the Fibonacci sequence.

Standards Addressed:

- * Mathematics standards addressing the development of number and number relationships (NCTM Standard 5).
- * Mathematics standards addressing number systems and number theory (NCTM Standard 6).
- * Mathematics standards addressing the exploration of patterns and function (NCTM Standard 8).
- * Mathematics standards addressing the study of the geometry of one, two, and three dimensions in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

More work on Fibonacci numbers and exploration of the Golden Section (page 10 in Zome System Manual, "Finding Tau," page 20 in Zome System Manual, and "Fibonacci Numbers and the Golden Section," "The Golden Thread - A History of Tau," pages 21-23 in Zome System Manual, and "Similarity and the Golden Section").

0	1	1	2	3	5	8	13	21
34	55	89	144	233	377			
610	987	1597	2584....					

Zome System

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Fibonacci Numbers and the Golden Section

Mathematics Intermediate Concept

Lesson Objective:

Students will investigate the relationships between the additive properties of **Fibonacci numbers** and the additive properties of the **Golden Section**, and discover the connection between the two.

Prerequisite Skills:

Experience working with Fibonacci Sequences (“Multiple Reflection Symmetry,” “Fun Fibonacci,” and page 10 in Zome System Manual). Basic knowledge of the properties of the Golden Section (“Finding Tau,” page 20 in Zome System Manual, and “The Golden Thread - A History of Tau”).

Time Needed:

One or two class periods of 45-60 minutes

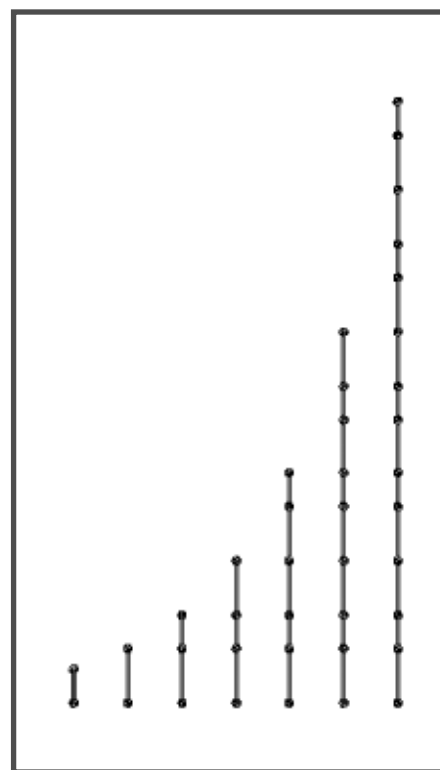
Materials Needed:

- Two Zome System Creator Kits for 25-30 students
- Graph paper
- Calculator (one per 5 students)

Procedure:

Divide the class into teams of 5 students, and distribute several Zome System nodes and struts of all three lengths in one color to each team. Review the concept of geometric progression from the class “Finding Tau”. In this progression of the golden section, two previous terms in the series add up to the next term in the series. *How can this be demonstrated in the lengths of the struts you have?*

Let the groups explore the struts for a few minutes to see if they can establish the progression in the lengths. Comparing the lengths of the longest strut with the short and medium added together can demonstrate the beginning of this series (short strut + medium strut = long



Fibonacci Numbers and the Golden Section

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strut). *What would be the next step in this series?* In the “Finding Tau” lesson, we learned that $X + 1 = X^2$ for the golden section, and that $t^2 = t + 1$.

Have the students continue the series by using the adding rule. *What is the pattern developing in this series?* The name of this pattern is the Fibonacci Sequence. *How do these numbers relate to the Golden Section? Is there any similarity to how they develop? How does this series differ from the Golden Section? Which one contains whole numbers? What kind of numbers does the Golden section contain?* If the two series are connected in the way they add the two previous values to obtain the third, there will be a connection between the ratios of adjacent numbers. Ask the students to see if they can discover what happens to the value of the ratios of adjacent numbers as the series grows, using graph paper and calculators. Again, allow a few minutes for the teams to explore the relationships on graphs.

The clearest way display the relationships is to draw a graph on the board and mark the value of t (1.6180339...) as a horizontal line above the x-axis. Ask students to plot the successive ratios of adjacent Fibonacci numbers, and mark them in the same graph. Thus the first ratio is $1/1=1$, the second is $2/1=2$, the third is $3/2=1.5$, the fourth is $5/3=1.666...$, $8/5=1.6$, $13/8=1.625$, etc. Students will see how the values alternates above and below the value of the Golden Section. The ratios rapidly converge on the exact value of t , and the connection between these series becomes clear.

The teams should both discuss mathematical and philosophical implications the their findings, and note their thoughts in their math journals.

Assessment:

Review the graphs created by the teams, as well as notes in individual math journals. To meet the standards, students must be able to show the similar additive property of the Golden Ratio lengths of the Zome System struts, and in the Fibonacci Sequence. They must also note that the Fibonacci Sequence and the Golden ratio differ in that the former consists of whole numbers, the latter is made up of irrational numbers. To exceed the standards their graphs must be plotted to show how successive ratios of adjacent Fibonacci numbers converge toward the Golden Section.

Standards Addressed:

- * Mathematics standards addressing **the development of number and number relationships** (NCTM Standard 5).
- * Mathematics standards addressing **number systems and number theory** (NCTM Standard 6).
- * Mathematics standards addressing the **exploration of patterns and function** (NCTM Standard 8).

Transfer Possibilities:

More advanced work with the Tau notation (pages 21-23 in Zome System Manual, and “Similarity and the Golden Section”).

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Archimedean Solids

Mathematics / Philosophy Advanced Concept

Lesson Objective:

Students will explore **semi-regular**, or **Archimedean** solids. They will determine how many such solids exist, which of them can be built with Zome System, and they will build one of them in its entirety.

Prerequisite Skills:

Knowledge of basic polygons (“Geometric Shapes”), and ability to define a two-dimensional versus a three-dimensional figure (“2-D and 3-D Shapes”). Experience building geometric solids (“Plato’s Solids - I,” “Plato’s Solids - II,” and “2-D and 3-D Stars”).

Time Needed:

Two class periods of 45 to 60 minutes.

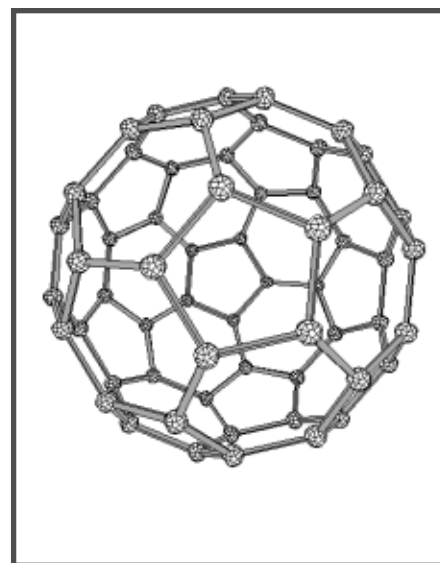
Materials Needed:

- Two or three Zome System Creator Kits for 25-30 students, or two kits and extra sets of blue struts.
- Four to six packs of supplementary green Zome System struts, if available
- Card stock polygons from Resource section of this document
- One pair of scissors per team
- One roll of tape per team

Procedure:

Prepare for the class by producing a large number of regular paper polygons. An easy way to do this is to enlarge the shapes in the resource section 200-300%, and cut them out. The paper polygons will be easiest to work with if they are copied onto, or glued onto, slightly heavier paper or card stock.

Start the class with a brief review of polyhedra. *Are polyhedra always made up of polygons? How are polyhedra named?*



Face angles of polygons

triangle	60°
square	90°
pentagon	108°
hexagon	120°
octagon	135°
decagon	144°

What is a **regular polyhedron**, or **Platonic solid** (convex figures where all faces, edge lengths, and angles are identical)? How many such shapes exist (5)? Which polygons are they made up of? What are they called? In this lesson the students will explore another family of polyhedra called semi-regular, or Archimedean solids. Why would a polyhedron be called semi-regular (they are made up of more than one type of regular polygons and have identical vertices)?

Divide the class into teams of 3-4 students and distribute Zome System pieces, the scissors, tape, and card stock polygons. The task for the teams is to discover how many semi-regular polyhedra exist using paper polygons and Zome System. How should we go about determining how such solids exist? Is the number unlimited or are there a set number? Ask the students to write down their predictions in their math journals. Discuss the various strategies the students propose.

Vertices buildable with just blue struts

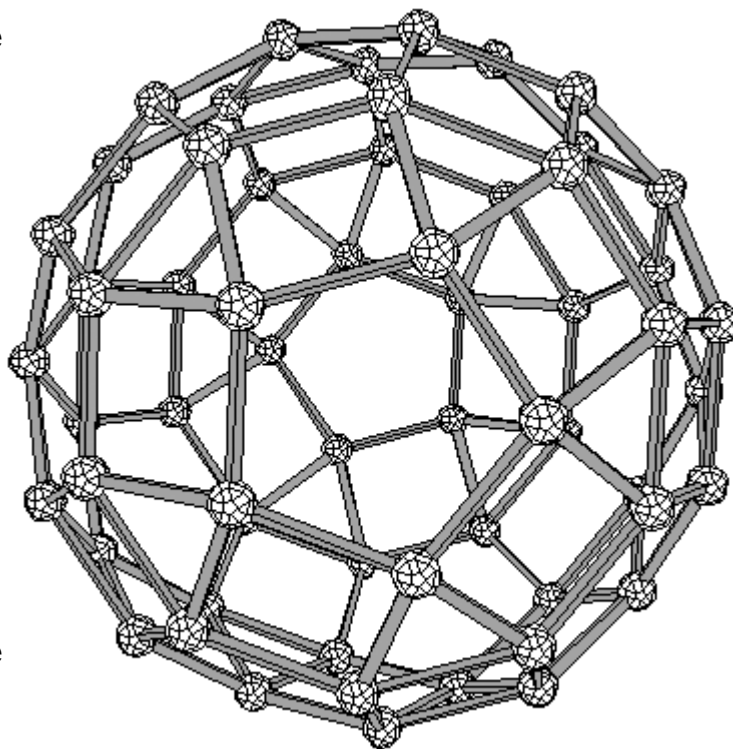
- 1 pentagon, 2 hexagons
- 2 pentagons, 2 triangles
- 2 decagons, 1 triangle
- 2 pentagons, 2 triangles
- 1 pentagon, 2 squares, 1 triangle

Vertices requiring green struts

- 2 hexagons, 1 triangle
- 2 squares, 2 triangles
- 3 squares, 1 triangle
- 2 hexagons, 1 square
- 1 octagon, 1 square, 1 hexagon
- 1 pentagon, 4 triangles

Vertices not buildable in Zome System

- 1 square, 4 triangles
- 1 pentagon, 4 triangles



One strategy is to first decide how many different ways polygons can be fitted together to create a polyhedron vertex.

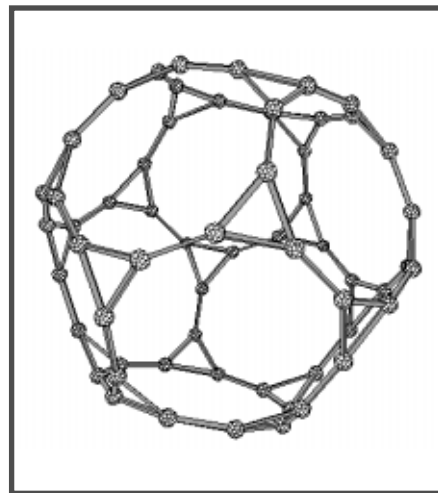
Zome System

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Students should keep in mind that vertices made up of identical polygons will build the 5 Platonic, or regular solids. To be a “legitimate” vertex the sum of the meeting face angles must be less than 360° . The student teams should build as many different vertices as they can find using the Zome System elements. They can supplement by also taping together cardstock polygons. Allow 25-30 minutes for this exploration. The student teams must make notes of all their findings in their math journals.

Go through the produced vertices with the whole class. *How many of the 13 possible vertices did the teams find? How many could be built with blue struts? How many require the green struts?* The 13 vertices are shown in the table on the right. Six of the vertices, and thus the solids, require the supplementary green Zome System struts. Two of the solids cannot be built with Zome System, and must be modeled with cardstock polygons.

The next task for the teams is to choose one of the vertices that can be built in Zome System, and make a model of the complete solid. They should learn the name and composition of this solid and introduce it to the class. The table below



Name of Solid	Faces
Cuboctahedron:	6 squares and 8 triangles
Great rhombicosadodecahedron	12 decagons, 20 hexagons, 30 squares
Great rhombicuboctahedron	6 octagons, 8 hexagons, 12 squares
Icosadodecahedron	12 pentagons, 20 triangles
Small rhombicosadodecahedron	12 pentagons, 20 triangles, 30 squares
Small rhombicuboctahedron	18 squares, 8 triangles
Snub cube	6 squares, 32 triangles
Snub dodecahedron	12 pentagons, 80 triangles
Truncated cube	6 octagons, 8 triangles
Truncated dodecahedron	12 decagons, 20 triangles
Truncated icosahedron	12 pentagons, 20 hexagons
Truncated octahedron	6 squares, 8 hexagons
Truncated tetrahedron	4 hexagons, 4 triangles

can be written on the board, or given as a handout to the students. *What can we notice about the number of the different polygons in the solids? Which numbers are repeated? How do the numbers of different polygons relate to the regular, Platonic, solids?*

The solids listed were first described by the Greek mathematician and philosopher Archimedes (c. 287-212 BC). The Archimedian solids are not common in nature. However, the truncated icosahedron is the shape of the C_{60} carbon molecule or “Buckyball”. Robert Curl and Richard Smalley of Rice University in Texas, and Harold Kroto of Sussex University in England, were awarded the 1996 Nobel Prize in chemistry for the discovery of this molecule.

An interesting extension activity is to test Euler’s formula against the models built by the students.

Assessment:

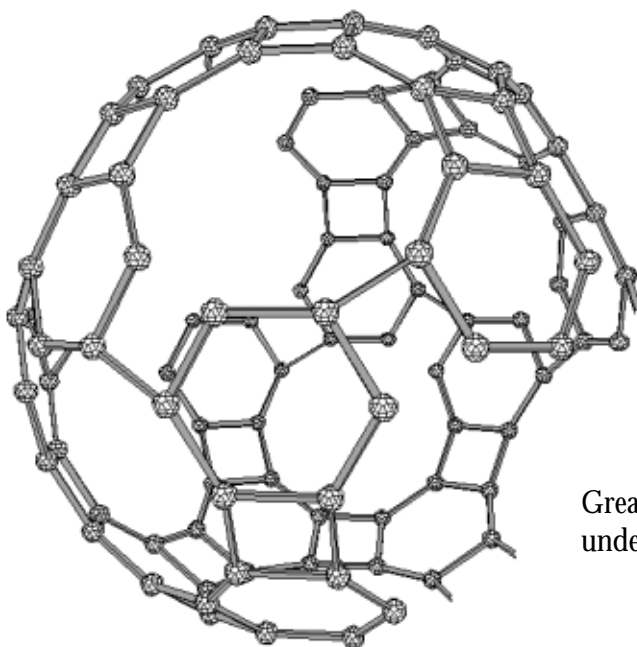
Observe and students as they build their structures, and take notes of their findings. Review notes in math journals. To meet the standard, students must complete at least 5 of the vertices using either Zome System elements or cardstock polygons. To exceed the standard they must identify all 13 possibilities, and determine how many can be built with Zome System.

Standards Addressed:

* Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12).

Transfer Possibilities:

Continued exploration of Polyhedra shapes (constructions 4, 5, 6, and 8 in Zome System Manual). More work on three-dimensional tessellations (“3-D Triangle Tiles” and “Beehive City”).



Great Rhombicosadodecahedron
under construction.

Mathematics Advanced Concept

Lesson Objective:

Students will discover why the shape of the bee's honeycomb is derived directly from the geometry of densest packing of spheres in three dimensions.

Prerequisite Skills:

Knowledge of basic polygons ("Geometric Shapes"), and ability to define a two-dimensional versus a three-dimensional figure ("2-D and 3-D Shapes"). Ability to build and name polyhedra shapes ("Plato's Solids - I," "Plato's Solids - II," and "Naming 2-D and 3-D shapes").

Time Needed:

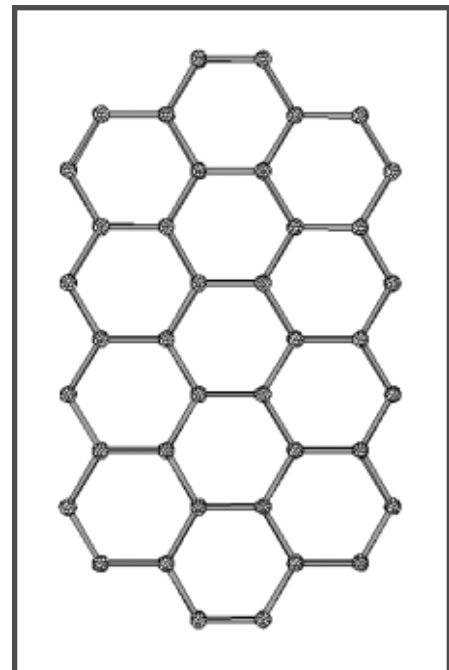
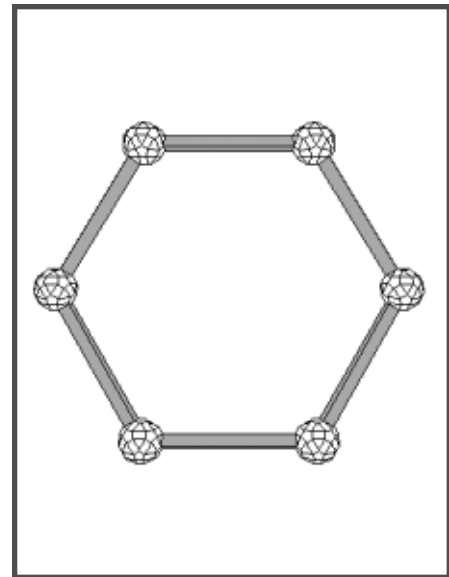
Two class periods of 45 to 60 minutes

Materials Needed:

- Three Creator Kits or 2 Creator Kits plus two bags of yellow struts
- Pennies, about 50 cents per team
- Ping Pong balls, about 10 per team
- Card stock grids of diamonds to cut out, see Resources section in back of this document
- Scissors, tape, about 1 set per team
- All-purpose glue (one container per team)
- Real bee honeycomb (one for each team if possible)

Procedure:

Begin by inviting the students to discuss honeycombs. *In what shapes do bees make their honeycombs? Does anyone know why all bees have honeycombs in this shape?* Lead a discussion on the efficiency of nature and its structures. *Is there any example where nature wastes energy in accomplishing a structure, pathway, or task?* Let students list as many examples they can think of in which nature conserves energy and maximizes efficiency. If no already listed, you might suggest:



Beehive City

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surfaces in bubbles, packaging of seeds, dormancy in winter, flow of a river, crystals in rocks and minerals, and the large ears on rabbits as temperature controls.

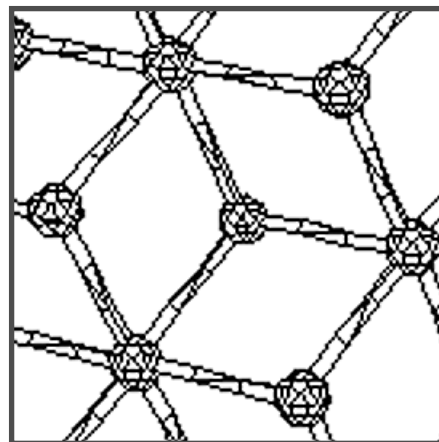
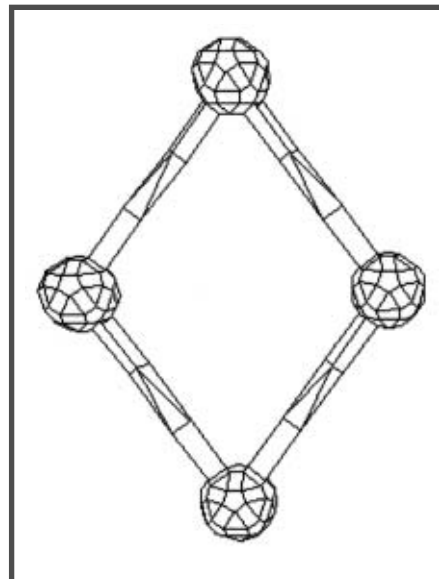
What is the purpose of the honeycomb? The purpose is dual; the honeycomb serves as an incubation chamber for growing worker bees, as well as to store honey, which is their food for the winter. So, it makes sense that they would want to produce the most storage chambers in the given amount of space.

What sort of spacing would provide this? How many other cells should each cell touch? What would be a good method of discovering the spacing? Distribute the pennies to the teams, and let them experiment with different spacing combinations. *What is the most number that can touch each other without overlapping? What is the least? In which configuration can the most pennies fit into a space, without overlapping? How can we draw lines to get rid of the gaps in between the pennies?* Once the teams have found the most effective grid structure, they should build a copy using blue Zome System struts. This is called a hexagonal packing structure.

Bees live in three dimensions. Using the ping pong balls and glue, repeat the exercises. *In what way can you get each ball to touch the least other balls? How about touching the most?* Moving in three dimensions, count the balls that touch (nearest neighbors). One way to accomplish this is to start with one ball, and see how many spheres can be glued to that central sphere. Another way is to glue together the closest packing formation in two dimensions, stack a closest packing layer on top of that and glue it, and when the glue is dry, glue another layer below that. Each team should have created the closest packing of spheres.

How can we apply the hexagonal structure from the penny model to the three dimensional version? Instead of passing lines through the points of contact between the pennies, pass a plane through the points of contact between ping pong balls.

What would this look like? Can you describe the shape that would be formed? How can we observe this? If the class cannot visualize the shapes, have them use the cardstock with diamonds copied onto it. Cut out the shapes, and fit them



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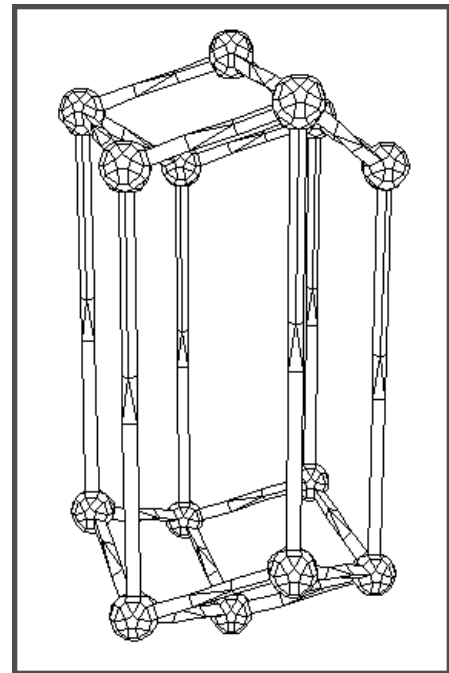
together. This shape would have 12 identical rhombic faces. It is called a **rhombic dodecahedron**. This shape can fill space with copies of itself. Challenge each group to build a rhombic dodecahedron in Zome System. *What connection does this have with the honeycomb?*

Divide the honeycomb into pieces, one for each group, or distribute the pieces. *What does the bottom surface of the comb look like? What is on the other side? Is a honeycomb made up of just hexagons?* Discuss how the bottom of the comb is divided into 3 equal diamonds, which come together at angles. Compare it with the rhombic dodecahedron. *What is similar between the two?* This diamond has diagonals in a ratio of $1:\sqrt{2}$. *How can we make a complete Zome System model of a bee honeycomb? How can we change the rhombic dodecahedron to make it resemble a honeycomb?* Give the teams time to find that they must use short and long yellow sticks to build elongated rhombic dodecahedrons to form a long hexagonal column with triads of diamonds. They must remove the triad on one of the ends. Several of these columns can then be packed together to form a honeycomb.

How should we build the other side of this? Look at the real honeycomb to see how two columns, one from each side, fit together base to base. *How would we go about fitting in the other side of chambers? How does the position of columns on one side compare to the position on the other side? How do they relate? How many bee chambers fit into a cubic inch of real honeycomb?* Complete the lesson with a short review of the effectiveness of the honeycomb as a packing system in three dimensions.

Assessment:

Question teams as they work, and review their math journals. To meet the standard, students must determine which shapes provide the most effective packing in two and three dimensions. To exceed the standard they must build a Zome System honeycomb and verbalize a rule of why this structure meets the challenge faced by the bees.



Beehive City

Zome System

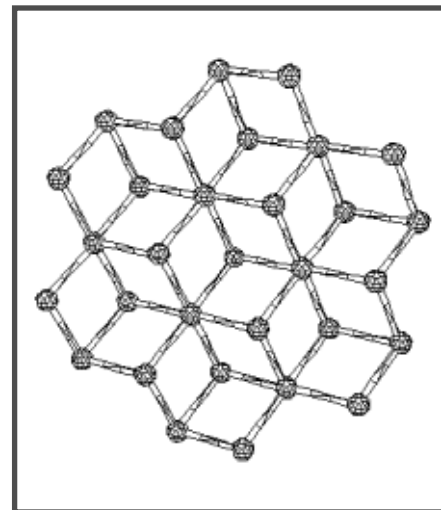
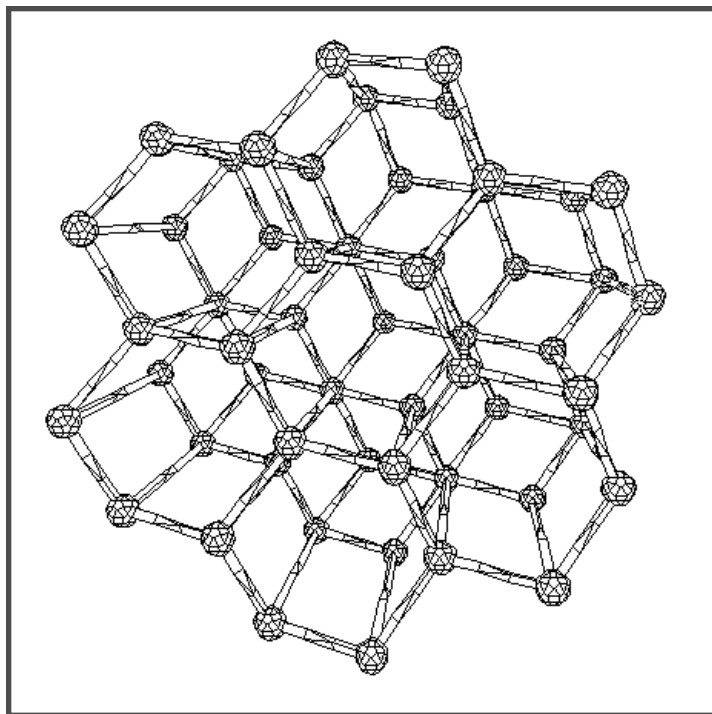
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Standards Addressed:

- * Science standards relating to biological structures, energy use in nature, and animal behavior.
- * Mathematics standards addressing the **extension of problem solving** (NCTM Standard 1).
- * Mathematics standards **addressing the continued study of the geometry of two and three dimensions** (NCTM Standard 7).

Transfer Possibilities:

Continued exploration of Polyhedra shapes (“Archimedean Solids”, and constructions 4, 5, 6, and 8 in Zome System Manual).



Zome System

Builds Genius!

Similarity and the Golden Section

Mathematics Advanced Concept

Lesson Objective:

Students will learn how the algebraic relationships contained in the golden section can be directly derived from a geometric relationship called similarity.

Prerequisite Skills:

Knowledge of the quadratic equation. Some background on the Golden Section (“Finding Tau,” and “The Golden Thread-A History of Tau”).

Time Needed:

One or two class periods of 45-60 minutes.

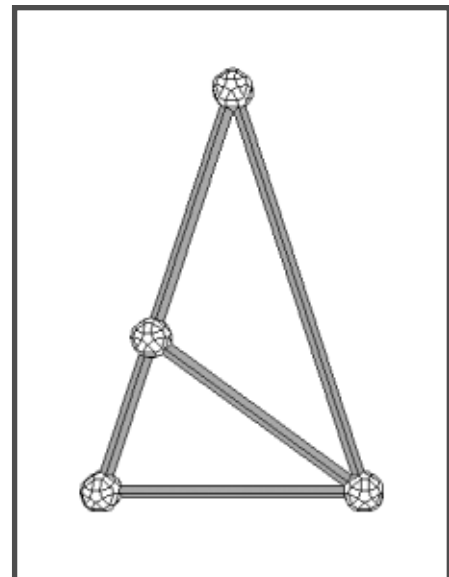
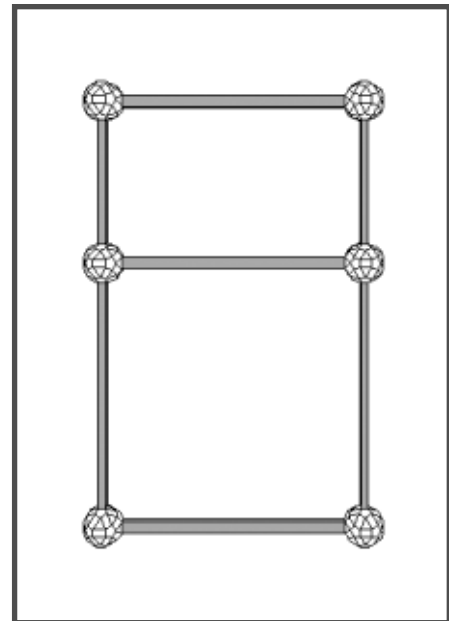
Materials Needed:

- One Zome System Creator Kit for 25-30 students

Procedure:

Prepare by drawing on the board the two shapes shown in the graphic. Indicate only the 108° angle of the triangle. Introduce the class by informing the students that they are going to algebraically derive the Golden Section using the concept of similarity. Direct their attention to the shapes on the board. *What are the shapes? How are they divided? Is there a relationship between each shape and its division?* Working in pairs, students should build Zome System models of the two shapes, using blue struts. They should also draw the triangle on paper and label the sides and angles. If it is given that the length of the long segment of the divided leg is x , and the short segment is 1, they should label the other sides using deduction. Allow the students a few minutes to complete this, and then go through the deduction with the whole class, as follows.

First, from the given information, the full length of the divided leg is $X+1$. Therefore, the other long leg is $X+1$ because they are the two similar legs of an isosceles trian-



Similarity and the Golden Section

Zome System

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gle. Second, the inner line is the same length as the upper divided section since the upper triangle is also an isosceles triangle, so its length is X . And finally, the base of the whole figure must be the same as the inner line since the lower triangle is also an isosceles triangle, so its length must be X .

Ensure that each student follows this reasoning, and has labeled all sides correctly, as shown.

Next, have the students rebuild the triangles, separated for clarity as shown.

Now repeat the entire process using the divided rectangle. Review with the class how we can proportion from ratios for both figures as shown.

$$\frac{1}{X} = \frac{X}{X+1}$$

Cross multiplying, we get

$$X^2 = X + 1$$

Give the pairs of students ten minutes to try to reach the Golden Section, via the quadratic equation:

$$X = \frac{1 + \sqrt{5}}{2}$$

Finally, go through each step with the entire class.

As an extension, discuss the relevance of the golden section in mathematics and fine art.

Assessment:

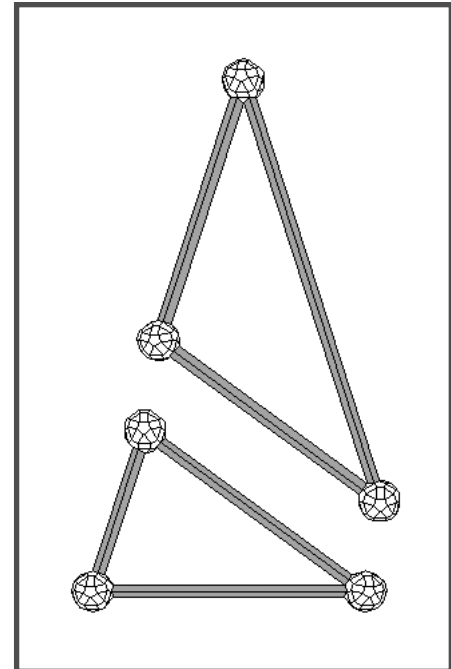
Question students about their deduction process, and review notes in their math journals. To meet the standard students must correctly label the lengths of the triangles and the rectangles, and follow the derivation of the golden section. To exceed the standard they must derive the section on their own.

Standards Addressed:

- * Mathematics standards addressing the continued study of algebraic concepts and methods (NCTM Standard 5).
- * Mathematics standards addressing the continued study of the geometry of two and three dimensions (NCTM Standard 7).

Transfer Possibilities:

Doing trigonometry and algebraic simplification with the Golden Section (pages 21-24 in "Zome System Manual").



Zome System

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Resources and Recommended Readings

This section lists various materials that can be used to support Zome System explorations. They are organized in the following order:

- Books Required to Teach Specific Plans
- Suggested Extension Resources
- Blueprint, or Photosensitive, Paper: How to Use With Zome System
- Polygon Shapes for Paper Models

Books Recommended for Specific Lesson Plans

Rogers, Paul, The Shapes Game. Henry Holt and Company, Inc., 1989. Suggested for the plan “Geometry Is All Around Us”.

Shapes, Ishtar Schwager Publications. Suggested for the plan “Geometric Shapes”.

Venezia, Mike, Getting to Know the World’s Greatest Artists: Picasso, Children’s Press, Chicago 1988. Suggested for the plan “Picasso and Math”.

Whitford Paul, Ann, Eight Hands Round, Harper Collins Publishers, 1991. Suggested for the plan “Trying Tessellation”.

Suggested Extension Resource

Zome System Specific:

Manual 2.0. Full-color brochure, included with all Zome System kits, that describes Zome System, including how it relates to numbers and symmetries in space, as well as building instructions for models.

Baer, Steve, Zome Primer, Zomeworks Corporation, Albuquerque, NM, 1970. Comprehensive exploration of the 31-zone structural system that forms the basis for Zome System. Included in the *Zome System Creator Kit*.

Booth, David. Zome System Manual, Parker Courtney Press, Chestnut Ridge, NY, 1996. Mathematics manual included in the *Zome System Explorer Kit* and *Zome System Creator Kit*.

Geometry and Symmetry:

Hargittai, István & Magdolna, Symmetry, Shelter Publications, 1994.

Beard, Col. R.S., Patterns in Space, Creative Publications.

Bentley, W.A. & W.J. Humphrey, Snow Crystals, Dover Publications, 1962.

Fibonacci Numbers and Numbers in Nature:

Hammel Garland, Trudi, Fascinating Fibonacci: Mystery and Magic in Numbers, Dale Seymour Publications, 1987.

Excellent posters illustrating Fibonacci patterns in nature, art and the built environment are available from Dale Seymour Publications, Delta Education, and several other educational publishers.

Golden Section:

Runion, Garth E. The Golden Section, Dale Seymour Publications, 1990

Posters illustrating the Golden Section and its mathematical and cultural relevance are available from Dale Seymour Publications, Delta Education, and other educational publishers.
Donald in Mathemagic Land, Walt Disney Productions, 1959. This easy-to-understand color film is available from many video outlets and libraries.

Blueprint or Photosensitive Paper: How to Use With Zome System

Blueprint, or photosensitive paper is fascinating to use to create permanent images of shadows cast by Zome System structures. This type of paper frequently comes in 24"x36" sheets or rolls. It can be obtained at hobby shops, book stores specializing in educational materials, and in some camera stores.

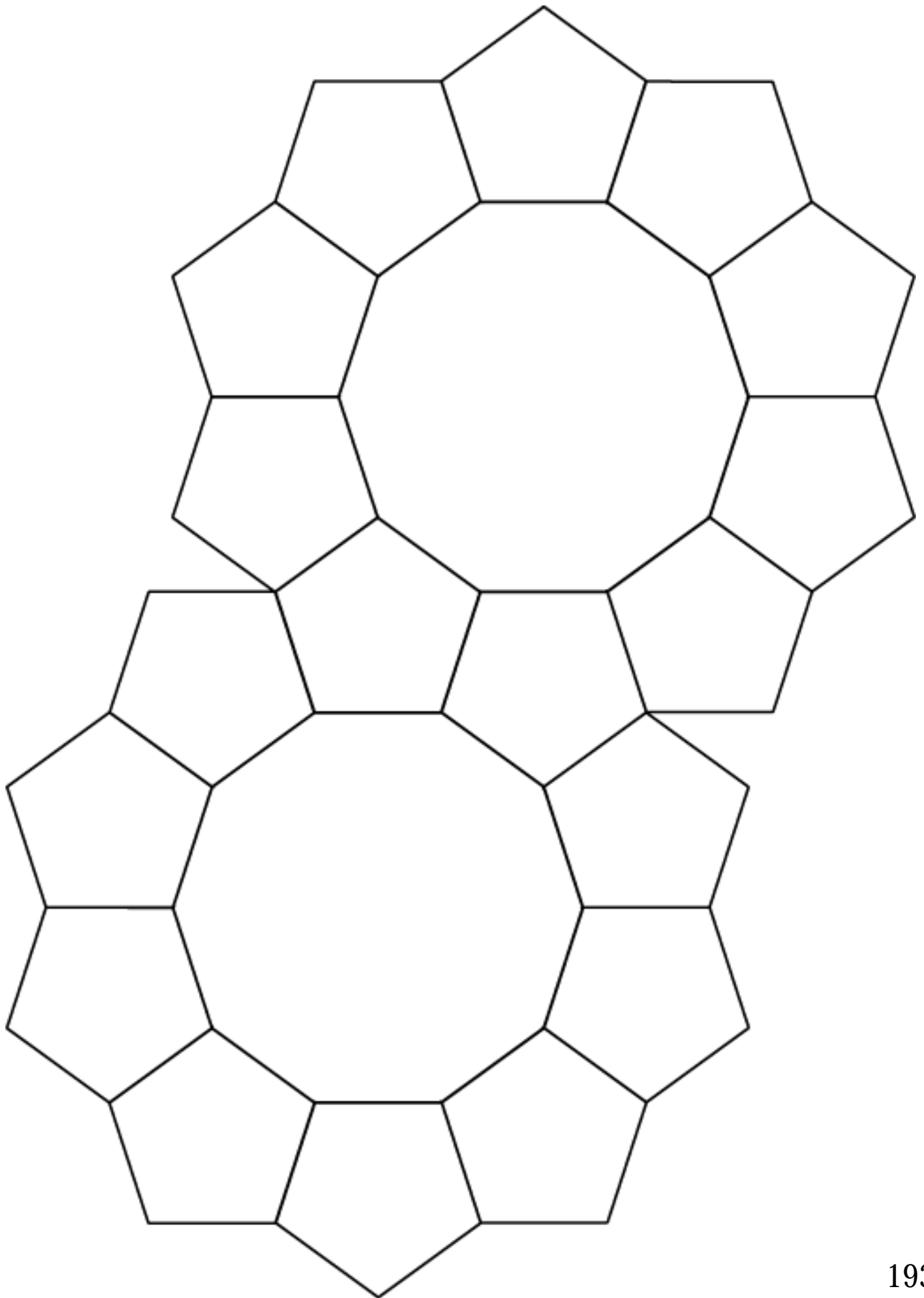
The paper should be stored in a light-safe container such as a thick black plastic bag. As the paper is "exposed" by sunlight, preparation work should take place in a work area with covered windows. Blueprint paper will be exposed by fluorescent light in less than an hour, incandescent light will take several hours. Return any paper that is not being used to the light-safe container.

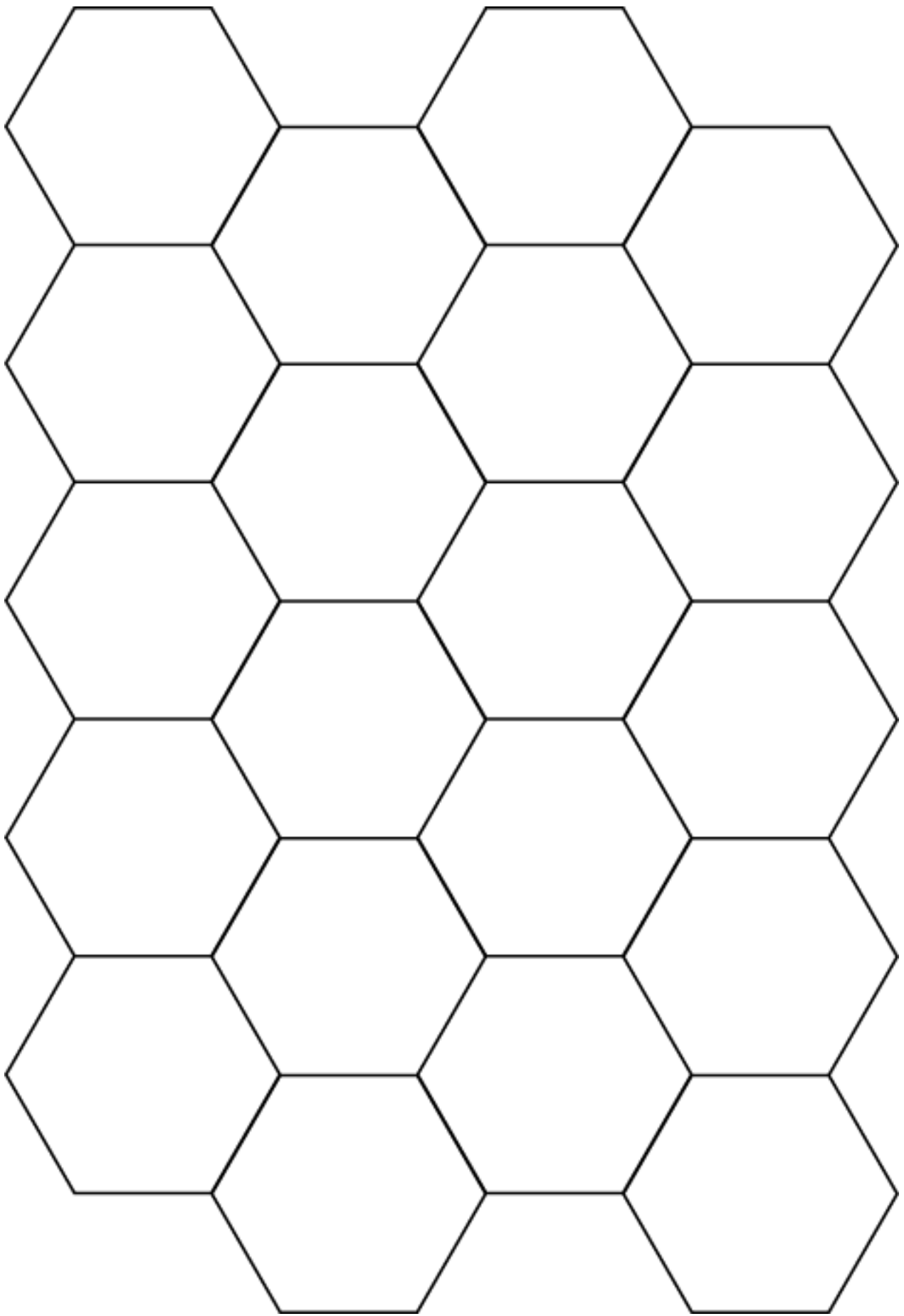
The supplier of the "blueprint" paper will have specific instructions on how to manipulate and "develop" the various brands of paper. The general procedure is as follows: Fix the paper to a hard backing such as a piece of cardboard with the light-sensitive yellow side of the paper facing up. The white side is not light-sensitive and will not yield any images. When the paper is exposed to sunlight for a 3-5 seconds, uncovered areas of the paper will rapidly fade from yellow into white. Pour approximately a cup of household ammonia solution in a non-aluminum cake pan or plastic tub. "Develop" the images by placing the mounted blueprint paper face down over the pan or tub for about 3 minutes. Areas of the paper that were not exposed to light will be colored a deep purple, exposed areas will give a white or light blue background.

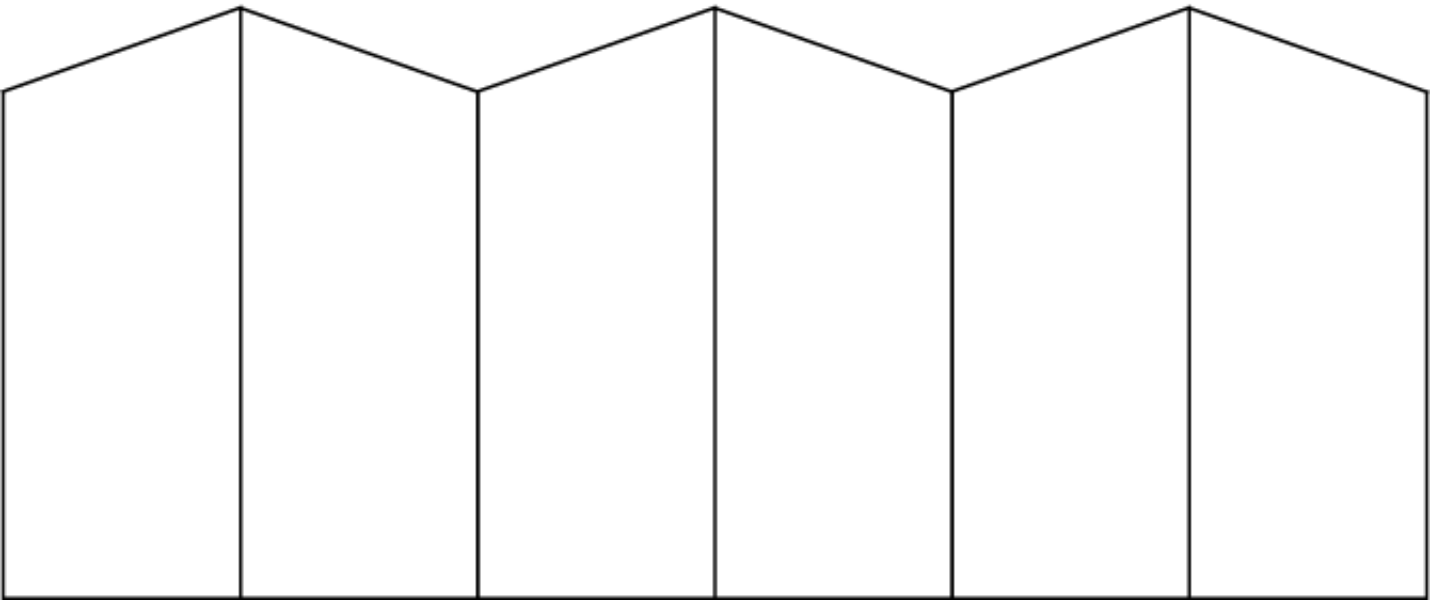
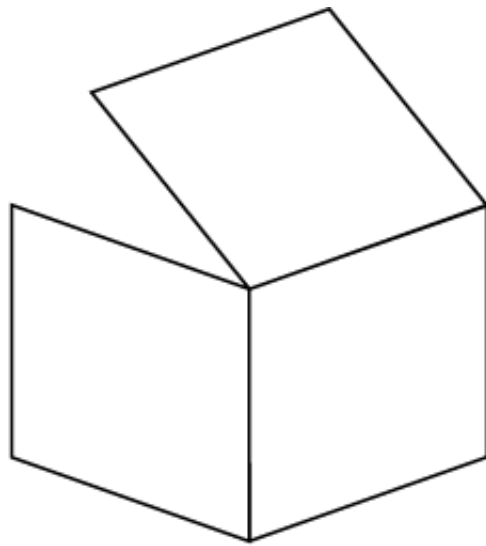
***CAUTION!!** ammonia solution is harmful if swallowed, irritating to the skin and the vapors are annoying. **FIRST AID:** Eyes; flush 10-15 minutes with water. Call a physician. Skin; flush thoroughly with water. Internal; immediately give large amounts of water or milk. Do not induce vomiting. Call a physician.*

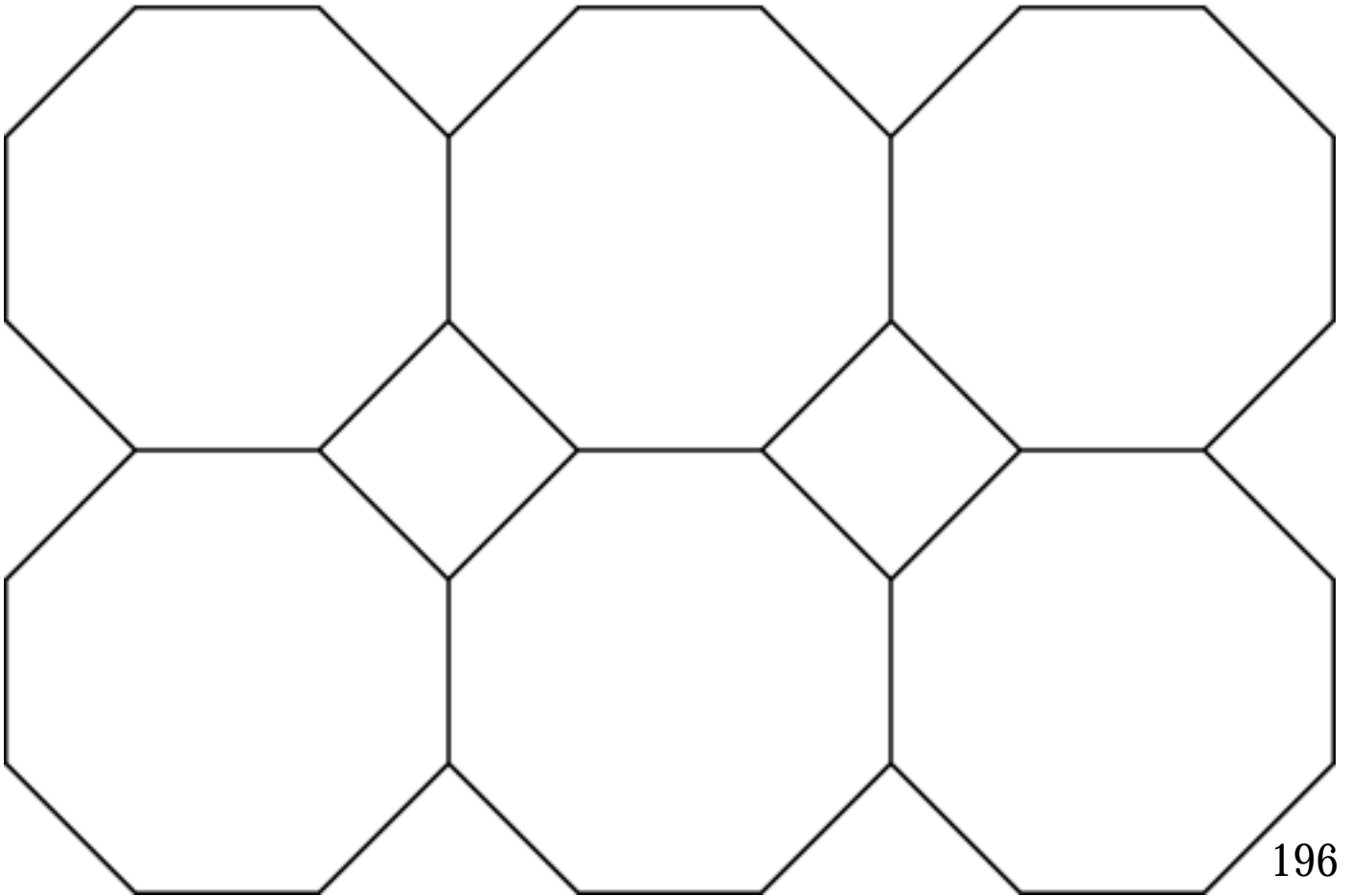
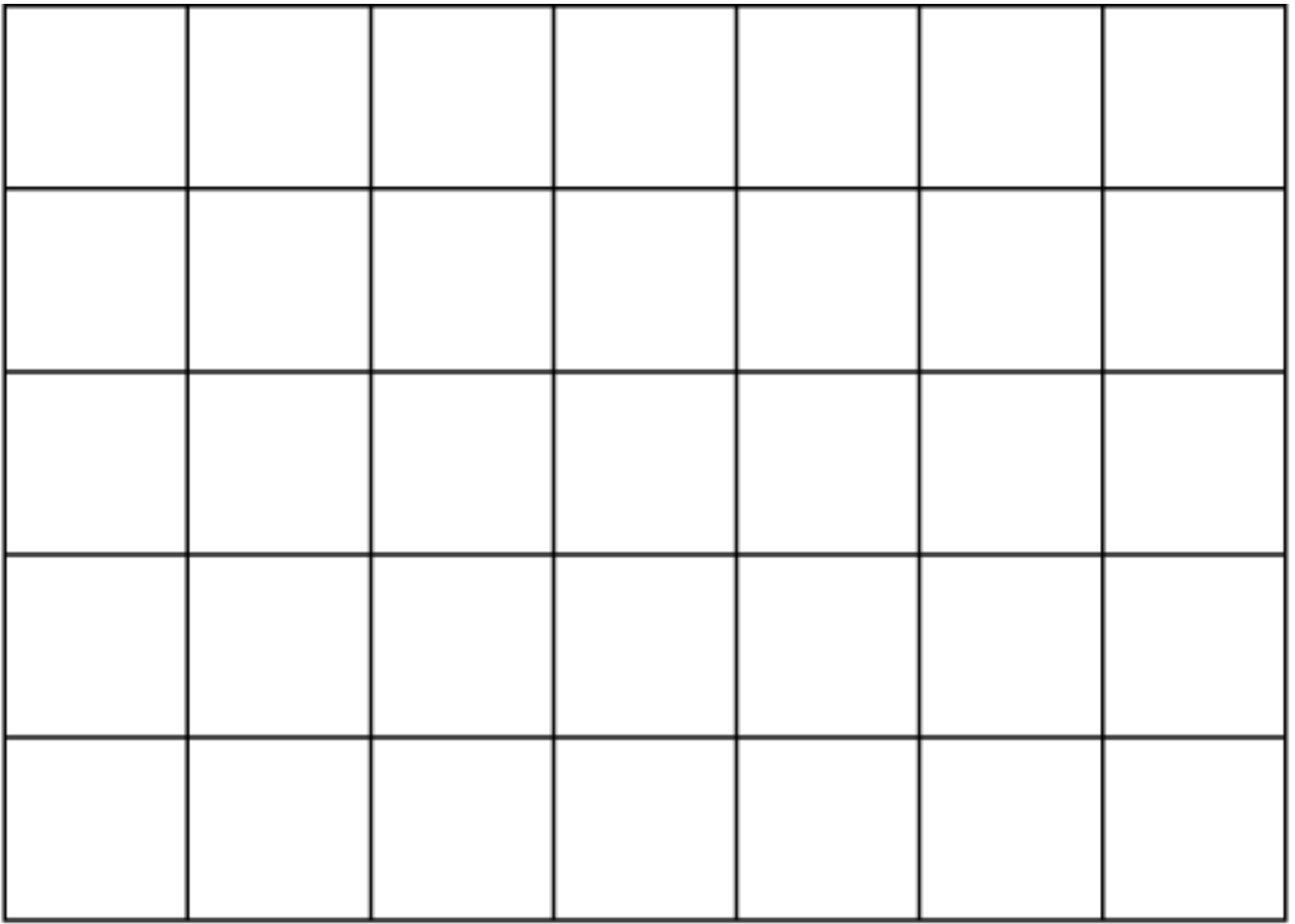
Polygon Shapes for Paper Models

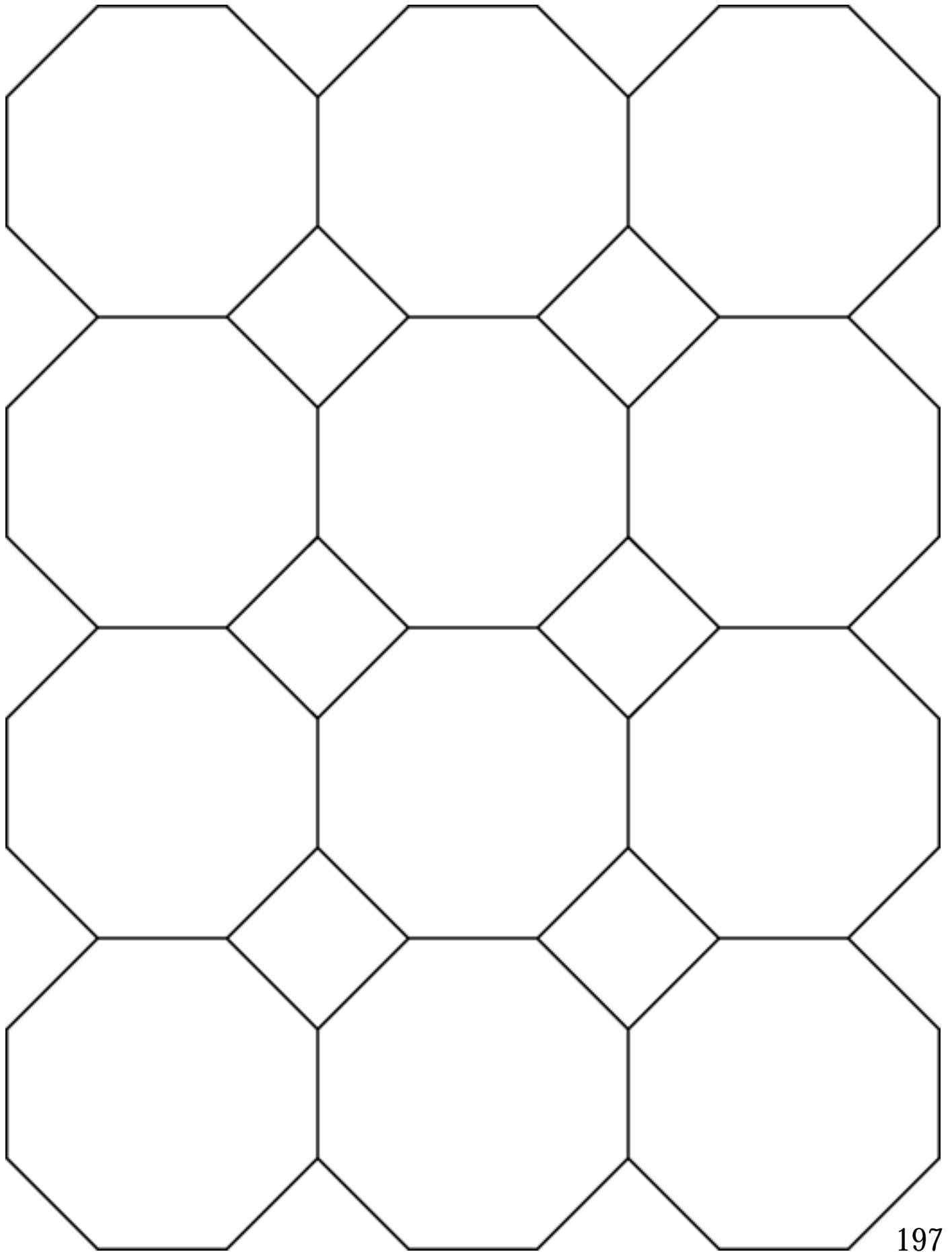
The following pages provides a variety of polygon shapes, most of which are equilateral. The shapes can be enlarged as appropriate using a photocopier. Transfer the polygon onto heavier paper and cut out. Plans that require these paper polygons include "Plato's Solids - I," "Plato's Solids - II," "Archimedes' Solids," and "Beehive City".

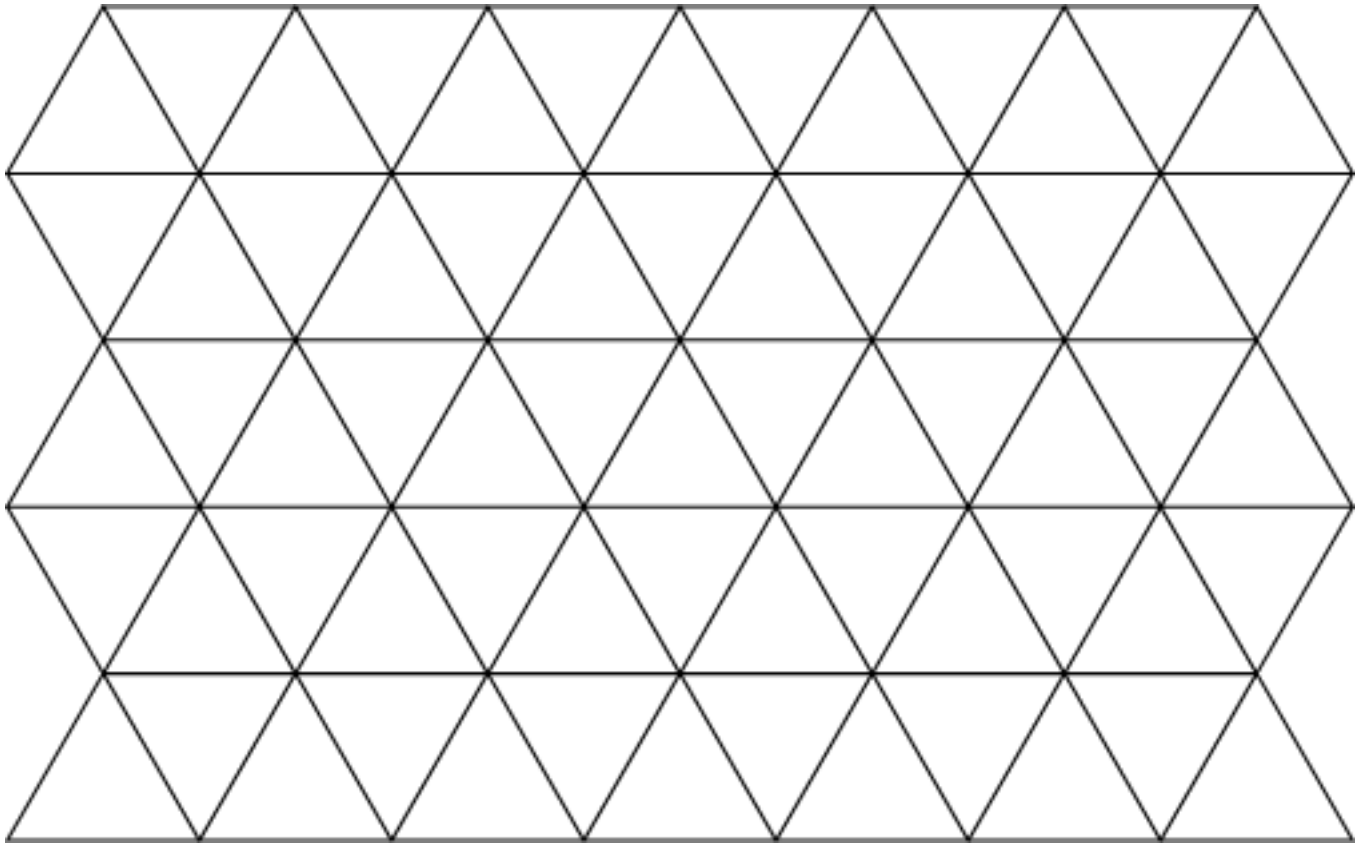


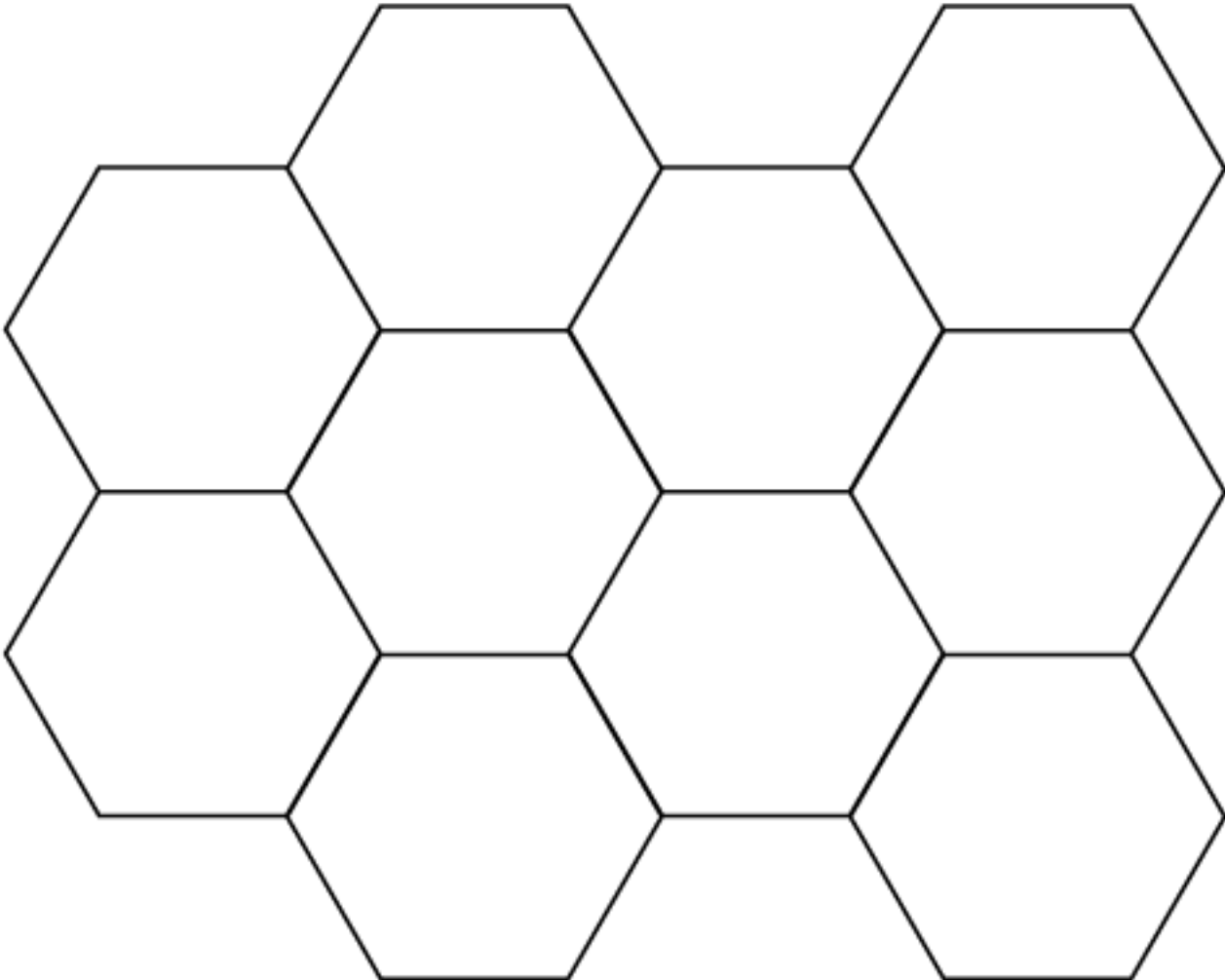
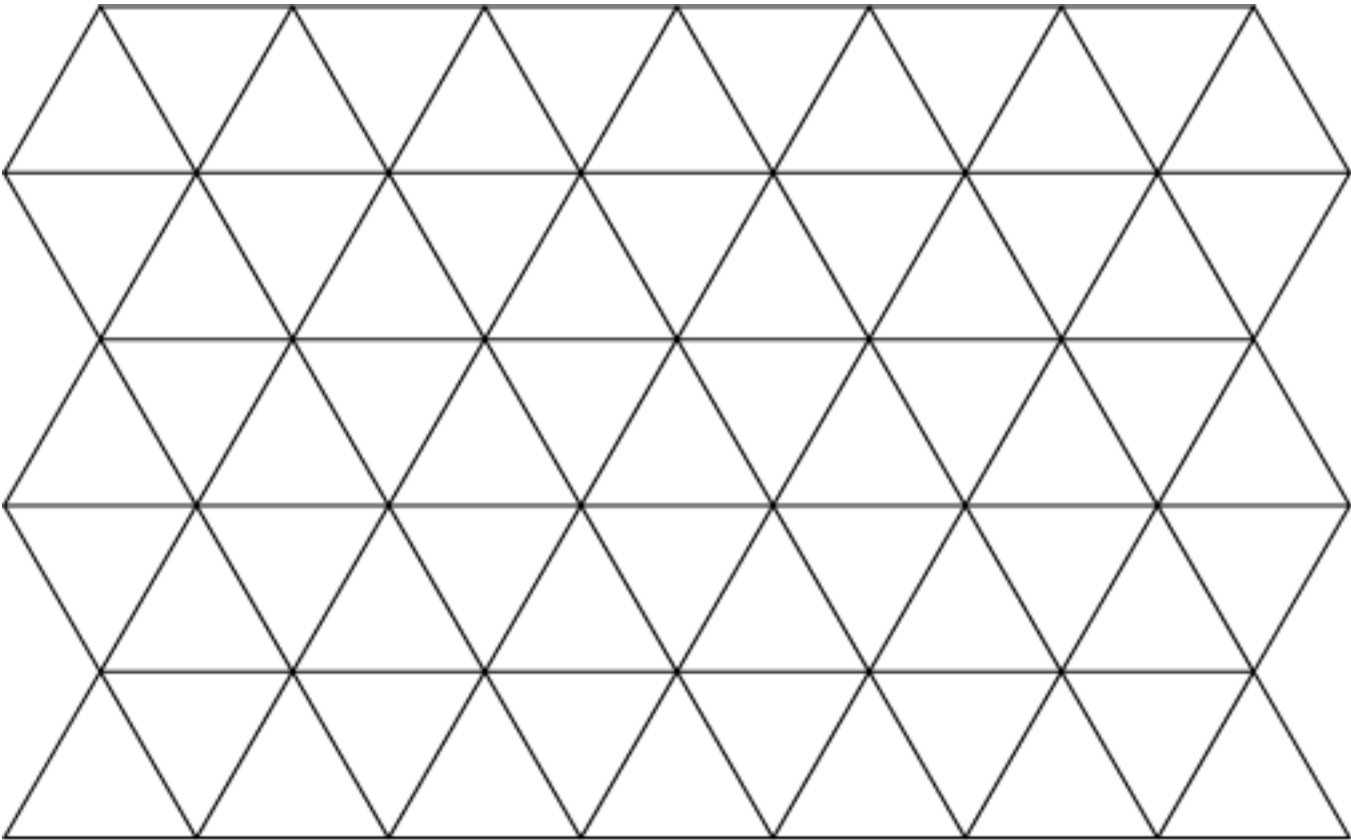












Zome System in Standards Based Education

The lesson plans make reference to the educational standards they address. Due to space constraints it was only possible to refer to the title of each standard. This section contains more extensive text, including the various benchmarks the plans address. The mathematics-related standards are based on those developed by National Council of Teachers of Mathematics (NCTM). Most US states have based their standards on the NCTM documents. The art and science standards have very general language borrowed from the standards used in the Jefferson County, Colorado, school district. These standards will be substantially re-written in later versions of the lesson plans.

Fine Art Standards

- * Fine Arts standards that **identify and apply the elements of art in a variety of media**. Addressed components include:
 - Identify, define, and practice the elements of art.
- * Fine Arts standards that **relate the visual arts to historical, cultural and personal heritage**. Addressed components include:
 - Creating works that are influenced by various cultures, and personal heritages;
 - Create art based on personal interpretation of historical and cultural context.

Language Arts Standards

- * Language Arts standards requiring students to **write and speak for a variety of purposes**. Addressed components include:
 - choosing vocabulary and figures of speech that communicate clearly;
 - writing and speaking in the content areas, using the technical vocabulary of the subject accurately.
- * Language Arts standards requiring students to **apply thinking skills to their reading, writing, speaking, listening, and viewing**. Addressed components include:
 - using reading, writing, speaking, listening, and viewing skills to solve problems and answer questions;
 - recognizing, expressing and defending a point of view orally in an articulate manner and in writing.

Physical Science Standards

- * Physical Science standards requiring students to **know and understand common properties, forms, and changes in matter and energy**. Addressed components include :
 - examining, describing, comparing, measuring, and classifying objects based on common physical properties;
 - developing simple models to explain observed properties of matter.

Mathematics Standards for Grades K-4

- * Mathematics standards addressing **mathematical connections** (NCTM Standard 4). Students are required to:
 - link conceptual and procedural knowledge;
 - relate various representations of concepts or procedures to one another;
 - recognize relationships among different topics in mathematics;
 - use mathematics in other curriculum areas;
 - use mathematics in their daily lives.

- * Mathematics standards addressing **number sense and numeration** (NCTM Standard 6). Students are required to:
 - construct number meanings through real-world experiences and the use of physical materials;
 - understand our numeration system by relating counting, grouping, and place-value concepts;
 - develop number sense;
 - interpret the multiple uses of numbers encountered in the real world;

- * Mathematics standards addressing **geometry and spatial sense** (NCTM Standard 9). Students are required to:
 - describe, model, draw, and classify shapes;
 - investigate and predict the results of combining, subdividing, and changing shapes;
 - develop spatial sense;
 - relate geometric ideas to number and measurement ideas;
 - recognize and appreciate geometry in their world.

- * Mathematics standards addressing **measurement** (NCTM Standard 10). Students are required to:
 - understand the attributes of length, capacity, weight, mass, area, volume, time, temperature, and angle;
 - develop the process of measuring and concepts related to units of measurements;
 - make and use estimates of measurement;
 - make and use measurements in problem and everyday situations.

Mathematics Standards for Grades 5-8

- * Mathematics standards addressing **mathematical problem solving as a method of inquiry and application** (NCTM Standard 1). Students are required to:
 - use problem-solving approaches to investigate and understand mathematical content;
 - formulate problems from situations within and outside mathematics;
 - develop and apply a variety of strategies to solve problems, with emphasis on multi-step and non-routine problems;
 - verify and interpret results with respect to the original problem situation;
 - generalize solutions and strategies to new problem situations;
 - acquire confidence in using mathematics meaningfully.

- * Mathematics standards addressing **mathematics as a means of communications** (NCTM Standard 2). Students are required to:
 - model situations using oral, written, concrete, pictorial, graphical, and algebraic methods;
 - reflect on and clarify their own thinking about mathematical ideas and situations;
 - develop common understandings of mathematical ideas, including the role of definitions;
 - use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas;
 - discuss mathematical ideas and make conjectures and convincing arguments;
 - appreciate the value of mathematical notation and its role in the development of mathematical ideas.

- * Mathematics standards addressing **mathematics as reasoning** (NCTM Standard 3). Students are required to:
 - recognize and apply deductive and inductive reasoning;
 - understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs;
 - make and evaluate mathematical conjectures and arguments;
 - validate their own thinking;
 - appreciate the pervasive use and power of reasoning as part of mathematics.

- * Mathematics standards addressing **investigation of mathematical connections** (NCTM Standard 4). Students are required to:
 - see mathematics as an integrated whole;
 - explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations;
 - use a mathematical idea to further their understanding of other mathematical ideas;
 - apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business;
 - value the role of mathematics in our culture and society.

- * Mathematics standards addressing **the development of number and number relationships** (NCTM Standard 5). Students are required to:
 - understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and scientific notation) in real-world and mathematical problem situations;
 - develop number sense for whole numbers, fractions, decimals, integers, and rational numbers;
 - understand and apply ratios, proportions, and percents in a wide variety of situations;
 - investigate relationships among fractions, decimals, and percents;
 - represent numerical relationships in one- and two-dimensional graphs.

- * Mathematics standards addressing **number systems and number theory** (NCTM Standard 6). Students are required to:
 - understand and appreciate the need for number beyond the whole numbers;
 - develop and use order relationships for whole numbers, fractions, decimals, integers, and rational numbers;

- extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers;
 - understand how the basic arithmetic operations are related to one another;
 - develop and apply number theory concepts (e.g. primes, factors, and multiples) in real-world and mathematical problem situations.
- * Mathematics standards addressing the **exploration of patterns and function** (NCTM Standard 8). Students are required to:
- describe, extend, analyze, and create a wide variety of patterns;
 - describe and represent relationships with tables, graphs, and rules;
 - analyze functional relationships to explain how a change in one quantity results in a change in another;
 - use patterns and functions to represent and solve problems.
- * Mathematics standards addressing **the exploration of algebraic concepts and processes** (NCTM Standard 9). Students are required to:
- understand the concepts of variable, expression, and equation;
 - represent situations and number patterns with tables, graphs, verbal rules, and equations, and explore the interrelationships of these representations;
 - analyze tables and graphs to identify properties and relationships;
 - develop confidence in solving linear equations using concrete, informal, and formal methods;
 - investigate inequalities and nonlinear equations informally;
 - apply algebraic methods to solve a variety of real-world and mathematical problems.
- * Mathematics standards addressing **the study of the geometry of one, two, and three dimensions** in a variety of situations (NCTM Standard 12). Students are required to:
- identify, describe, compare, and classify geometric figures;
 - visualize and represent geometric figures with special attention to developing spatial sense;
 - explore transformations of geometric figures;
 - represent and solve problems using geometric models;
 - understand and apply geometric properties and relationships;
 - develop and appreciation of geometry as a means of describing the physical world.
- * Mathematics standards addressing extensive **concrete experiences using measurement** (NCTM Standard 13). Students are required to:
- extend their understanding of the process of measurement;
 - estimate, make, and use measurements to describe and compare phenomena;
 - select appropriate units and tools to measure to the degree of accuracy required in a particular situation;
 - understand the structure and use of systems of measurement;

- extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass.
- develop the concepts of rates and other derived and indirect measurements;
- develop formulas and procedures for determining measures to solve problems.

Mathematics Standards for Grades 9-12

* Mathematics standards addressing **the extension of problem solving** (NCTM Standard 1). Students are required to:

- use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
- apply integrated mathematical problem solving strategies to solve problems from within and outside mathematics;
- recognize and formulate problems from situations within and outside mathematics;
- apply the process of mathematical modeling to real-world problem situations.

* Mathematics standards addressing **language and symbolism to communicate mathematical ideas** (NCTM Standard 2). Students are required to:

- reflect upon and clarify their thinking about mathematical ideas and relationships;
- formulate mathematical definitions and express generalizations discovered through investigations;
- express mathematical ideas orally and in writing;
- read written presentations of mathematics with understanding;
- ask clarifying and extending questions related to mathematics they have read or heard about;
- appreciate the economy, power, and elegance of mathematical notation and its role in the development of mathematical ideas.

* Mathematics standards addressing numerous and varied experiences that reinforce and extend **logical reasoning skills** (NCTM Standard 3). Students are required to:

- make and test conjectures;
- formulate counter-examples;
- follow logical arguments;
- judge the validity of arguments;
- construct simple valid arguments;

For college-intending students:

- construct proofs for mathematical assertions, including indirect proofs and proofs by mathematical induction.

* Mathematics standards addressing **the connections and interplay among various mathematical topics** and their applications (NCTM Standard 4). Students are required to:

- recognize equivalent representations of the same concept;
- relate procedures in one representation to procedures in an equivalent representation;

- use and value the connections among mathematical topics;
 - use and value the connections between mathematics and other disciplines.
- * Mathematics standards addressing **the continued study of algebraic concepts and methods** (NCTM Standard 5). Students are required to:
- represent situations that involve variable quantities with expressions, equations, inequalities, and matrices;
 - use tables and graphs as tools to interpret expressions, equations, and inequalities;
 - operate on expressions and matrices, and solve equations and inequalities;
 - appreciate the power of mathematical abstraction and symbolism;

For college-intending students:

- use matrices to solve linear systems;
- demonstrate technical facility with algebraic transformations, including techniques based on the theory of equations.

* Mathematics standards addressing **the continued study of functions** (NCTM Standard 6). Students are required to:

- model real-world phenomena with a variety of functions;
- represent and analyze relationships using tables, verbal rules, equations, and graphs;
- translate among tabular, symbolic, and graphical representations of functions;
- recognize that a variety of problem situations can be modeled by the same type of function;
- analyze the effects of parameter changes on the graphs of functions;

for college-intending students:

- understand operations on, and the general properties and behavior of, classes of functions.

* Mathematics standards addressing **the continued study of the geometry of two and three dimensions** (NCTM Standard 7). Students are required to:

- interpret and draw three dimensional objects;
- represent problem situations with geometric models and apply properties of figures;
- classify figures in terms of congruence and similarity and apply these relationships;
- deduce properties of, and relationships between, figures from given assumptions;

for college-intending students:

- develop and understanding of an axiomatic system through investigating and comparing various geometries.

* Mathematics standards addressing **the study of geometry of two and three dimensions from an algebraic point of view** (NCTM Standard 8). Students are required to:

- translate between synthetic and coordinate representations;
- deduce properties of figures using transformations and using coordinates;

- identify congruent and similar figures using transformations;
 - analyze properties of Euclidean transformations and relate translations to vectors;
- for college-intending students:
- deduce properties of figures using vectors;
 - apply transformations, coordinates, and vectors in problem-solving.
- * Mathematics standards addressing **the study of trigonometry** (NCTM Standard 9). Students are required to:
- apply trigonometry to problem situations involving triangles;
 - explore periodic real-world phenomena using the sine and cosine functions;
- for college-intending students:
- understand the connection between trigonometric and circular functions;
 - use circular functions to model periodic real world phenomena;
 - solve trigonometric equations and verify trigonometric identities;
 - understand the connections between trigonometric functions and polar coordinates, complex numbers, and series.
- * Mathematics standards addressing **the topics from discrete mathematics** (NCTM Standard 12). Students are required to:
- represent problem situations using discrete structures such as finite graphs, matrices, sequences, and recurrence relations;
 - represent and analyze finite graphs using matrices;
 - develop and analyze algorithms;
 - solve enumeration and finite probability problems;
- for college-intending students:
- represent and solve problems using linear programming and difference equations;
 - investigate problem situations that arise in connection with computer validation and the application of algorithms.
- * Mathematics standards addressing **the informal exploration of calculus concepts from both a graphical and numerical perspective** (NCTM Standard 13). Students are required to:
- determine maximum and minimum points of a graph and interpret the results in problem situations;
 - investigate limiting processes by examining infinite sequences and series and areas under curves;
- for college-intending students:
- understand the conceptual foundations of limit, the area under a curve, the rate of change, and the slope of a tangent line, and their applications in other disciplines;
 - analyze the graphs of polynomial, rational, radical, and transcendental functions.

* Mathematics standards addressing **the study of mathematical structure** (NCTM Standard 14).

Students are required to:

- compare and contrast the real number system and its various subsystems with regard to their structural characteristics;
- understand the logic of algebraic procedures;
- appreciate that seemingly different mathematical systems may be essentially the same;

For college-intending students:

- develop the complex number system and demonstrate facility with its operations
- prove elementary theorems within various mathematical structures, such as groups and fields;
- develop an understanding of the nature and purpose of axiomatic systems.

Acute angle: An angle whose measure is less than 90 degrees.

Angle: A pair of Zome System struts that share the same Zome System node but do not lie on the same line.

Archimedes: Greek philosopher and mathematician (287 BC-212 BC). He was the first person to describe the semi-regular solids.

Archimedean solid: Also known as **semi-regular solids**. A group of 13 polyhedra defined by the Greek philosopher Archimedes. Archimedean solids have regular polygon faces of two or more kinds and all alike vertices.

Area: A measure, in square units, of how much 2-dimensional space is occupied by a given 2-dimensional figure.

Axis: The line formed by any Zome System strut and the strut on the exact opposite side of the node.

Bilateral symmetry: The repetition of parts by a mirror reflection across the center line of given object.

Blue struts: Fit into the rectangular holes in the node. The blue struts are the “unity struts” which are used to build all regular polygons and polyhedra.

Blue line: Line created by blue struts. Structures built on planes perpendicular to a blue line will frequently display 2-fold and 4-fold symmetries.

Buckyball: Popular name for the Carbon 60 (C^{60}) molecule. The buckyball was discovered in the late 1980s and is the first new form of carbon to be created beyond the naturally occurring forms graphite and diamond. A Zome System model of a Buckyball is made by building a sphere consisting of 12 regular pentagons and 20 regular hexagons. The geometric name for this shape is truncated icosahedron.

Buttress: A structure projecting from a large expanse of wall. The buttress supports the wall from the outside. Gothic cathedrals used buttresses to strengthen walls weakened by huge stained glass windows.

Color coding: In addition to being coded according to shape the Zome System struts are also coded according to color. Rectangular struts are blue. Triangular struts are yellow. Pentagonal struts are red. Nodes are a neutral white.

Concave polygon: A polygon that has some interior vertex angles which are 180 degrees or more; i.e. one of the vertices “points in”.

Concave polyhedron: A polyhedron that has some dihedral angles which are 180 degrees or more.

Congruent, congruency: Two or more geometric figures are congruent if all their angles and edges and faces are exactly the same.

Convex polygon: A polygon that has all interior vertex angles less than 180 degrees.

Convex polyhedron: A polyhedron that has all dihedral angles less than 180 degrees.

Cross-section: Any slice, real or imaginary, through any object to see inside it.

Crystal: A solid state of matter where the atoms are in a periodic or a quasi-periodic arrangement. It is possible to build a wide range of crystal forms with Zome System. Examples of crystals include sodium chloride (table salt), quartz (silica), graphite (a form of carbon).

Crystalline lattice: The actual arrangement of atoms in a given crystal. If the structure is periodic there are 230 possible arrangements.

Cube: Polyhedron shape consisting of six square faces. The regular cube can be built in Zome System using blue struts. The cube, which is also known as a “hexahedron”, is one of the five Platonic solids.

Cuboctahedron: One of the 13 Archimedean (semi-regular) solids. It has 6 square faces and 8 triangle faces. Each vertex has 2 squares and 2 triangles. Requires the supplementary green struts to build.

Deca: A Greek prefix meaning 10.

Decagon: Ten-sided polygon. The equilateral decagon is built with blue struts.

Dodeca: A Greek prefix meaning 12.

Dodecagon: A polygon having 12 sides.

Dodecahedron: One of the 5 Platonic (regular) solids. It has 12 pentagon faces. Each vertex has 3 pentagons. A polyhedron with twelve faces. The regular dodecahedron has pentagonal faces, and is one of Plato’s solids

Dihedral angle: The inside angle between 2 adjacent (sharing an edge) polygon faces of a polyhedron.

Divine proportion: A different name for the “Golden Section”.

DNA strand: A piece of a spiral DNA molecule. DNA carries all the genetic information of every living thing on earth.

Edge: Lines (struts) which form the boundary of a polygon, polyhedron or a polytope.

Enneacontahedron: A polyhedron consisting of 90 rhombic faces of 2 different kinds. Sixty fat rhombi and 30 skinny rhombi, 90 faces in total. The enneacontahedron can be built in Zome System using one length of yellow strut.

Equilateral: Having lines (struts) of all the same length.

Euler, Leonhard: Swiss mathematician (1707-1783)

Euler’s formula: General formula that applies to all convex polyhedra: $F+V-E=2$, or $F+V=E+2$. Where F =number of faces, V =number of vertices, and E =number of edges. This relationship was first documented by the Leonhard Euler in 1752. Students can show that the formula always holds up by experimenting with Zome System.

Face: A polygon piece of a polyhedron.

Fibonacci sequence: A series of numbers where each number is the sum of the two numbers before it in the sequence. The entire sequence is 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144... ($0+1=1$, $1+1=2$, $2+1=3$, $3+2=5$, $5+3=8$, $8+5=13$...). Zome System struts also work in this sequence; short strut + medium strut = long strut, medium strut + long strut = next length strut...). Fibonacci numbers can be found in virtually all plants with spiraling patterns. For example, a pine cone has 8 spirals going in one direction, and 13 going in the other. The seeds of a strawberry also grow in spirals, 13 in one direction, and 21 in the other. Cauliflower florets combine in spirals of 5 and 8, and sunflowers have 34 and 55 spirals of seeds.

The Fibonacci sequence was first documented in the 13th century by the Italian mathematician Leonardo of Pisa, who was also known as Fibonacci (son of Bonacci).

5-Fold symmetry: Repeating a motif by 5 reflections or 5 rotations around a central point. Five-fold symmetries and clusters of five are very common in nature. Apples have a five-point star in the middle, starfish are five-point stars, lots of flowers and leaves have clusters of five petals or points. Most mammals have 5 fingers and toes. An acorn squash and a pumpkin has 10 sections (5×2).

Fractal progression: A pattern which repeats itself at different levels of size. For example, a cauliflower head displays fractal geometries in its spirals. The spirals are made of smaller spirals which in turn are made of even smaller spirals and so on. The famous Mandelbrot set is another example of a fractal progression. Zome System can build fractal star progressions in Golden sections.

Fuller, R. Buckminster: Famous 20th century American architect and inventor of the Geodesic dome. Fuller also designed many space frames (triangulated structures).

Fullerene: A family of laboratory-created molecules that includes C^{60} (Buckyball), and C^{240} . The name Fullerene was given because the structures of these molecules reminded researcher of the geodesic domes created by R. Buckminster Fuller. Zome System can be used to model a wide range of Fullerenes.

Geodesic dome: A symmetrical tiling of triangles covering a sphere. These structures are extremely strong and light, giving the maximum strength for the minimum amount of building material.

Geometric progression: A number sequence in which each successive term may be found by multiplying by the same number each time.

Golden rectangle: Rectangle built with short and medium, or medium and long blue struts. The sides of this rectangle are in the famous Golden section.

Golden rectangle grid: This structure shows the infinite spiral of squares contained in the Golden rectangle.

Golden section: Also known as the Divine Proportion, and as the Golden Mean. Irrational number noted with the Greek letter τ (tau) = 1.61802... The exact value can be calculated to any desired accuracy from the formula $\tau = \text{square root } 5 + 1/2$. Golden section proportions are very important in classical art and architecture. All Zome System struts are in golden sections of each other.

Great rhombicuboctahedron: One of the 13 Archimedean (semi-regular) solids. It has 6 octagon faces, 8 hexagon faces and 12 square faces. Each vertex has 1 octagon, 1 hexagon and 1 square. Requires blue and green struts to build in Zome System.

Great rhombicosadodecahedron: One of the 13 Archimedean (semi-regular) solids. It has 12 decagon faces, 20 hexagonal faces and 30 square faces. Each vertex has 1 decagon, 1 hexagon and 1 square. Can be built with blue struts.

Great stellated dodecahedron: A star polyhedron produced by placing 20 triangular pyramids on the 20 triangular faces of the regular icosahedron.

Green struts: These supplementary Zome System struts add 30 buildable angles while being fully compatible with all current components. Now available! For more information go to www.zomesystem.com.

Heptagon: A polygon having 7 sides.

Hexa: A Greek prefix meaning 6.

Hexagon: A polygon having 6 sides.

Hexahedron: Correct name for the cube, a polyhedron with six square faces.

Isosceles triangle: A triangle with 2 equal sides and 2 equal angles.

Icosa: A Greek prefix meaning 20.

Icosadodecahedron: One of the 13 Archimedean (semi-regular) solids. It has 12 pentagon faces and 20 triangle faces. Each vertex has 2 pentagons and 2 triangles. Can be built with blue struts.

Icosahedron: Polyhedron with 20 faces. The regular icosahedron has 20 equilateral triangles as faces. This shape is one of Plato's five solids. Can be built with blue struts.

Irrational: Any number with an infinite number of decimal places but no repeating pattern.

Kepler, Johannes: (1571-1630) German mathematician and astronomer. Discovered both stellations of the regular dodecahedron, the rhombic triacontahedron, the rhombic dodecahedron and non-periodic tilings. Kepler also devised a model of the solar system based on the relationship between the 5 regular solids to explain the relative distances of the planets from the Sun.

Kepler tilings: Periodic and non-periodic tilings of pentagrams, pentagons and decagons.

Keplerian solids: The edge stellated icosahedron and the edge stellated dodecahedron. These are starry pointed shapes created if we extend the edges an icosahedron or a dodecahedron.

Kite: A quadrilateral that has 2 pairs of edge lengths. The opposite edges are not equal length.

Length: A measure, in line segment units of how much 1 dimensional space is occupied by a given one dimensional object.

Line: A continuous set of points which is straight and extends without end in both directions.

Modularity: Having a small number of types of component parts which can be combined in many configurations.

Node: A Zome System connector ball. In the original Zome System kits the nodes are white. Zome System also manufactures colored nodes that are primarily used by various researchers working in chemistry and materials science.

Nomenclature: A standardized system of naming objects or processes.

Non-periodic: Having a motif which does not repeat by translations.

Obtuse angle: An angle of 180 degrees or more.

Octa: A Greek prefix meaning 8.

Octagon: A polygon having 8 sides.

Octahedron: A polyhedron with 8 faces. The regular octahedron, which consists of 8 equilateral triangles, is one of Plato's solids. Although it is possible to build a large number of octahedra with the regular Zome System struts, the regular octahedron requires the supplementary green struts.

Oct-tet truss: An infinite periodic network of regular octahedra and regular tetrahedra. When built as a structure of metal bars an extremely strong structure results. The first person to use this type structure was Alexander Graham Bell. Bell built kites with oct-tet trusses, including one large enough to carry a man aloft (well before the Wright brothers first flight!).

Parallel Lines: 2 lines are said to be parallel if they lie in the same plane and do not intersect.

Parallel Projection: To cast a shadow with a light source that is infinitely far away from the object.

Parallelogram: A quadrilateral in which opposite sides have equal lengths and are parallel.

Penta: A Greek prefix meaning 5.

Pentagon: A polygon having 5 sides.

Pentagram: A star shape created by extending (stellating) the edges of a pentagon.

Periodic: Having infinite repetition of the same motif in one continuous straight line direction.

Perpendicular line: A line at 90 degrees to another line.

Perspective projection: To cast a shadow with a light close to the object.

Phi, ϕ : One of two Greek symbols for the Golden section (Divine proportion). This one is used more by artists and people interested in the more esoteric aspects of the Golden section.

Plane: An infinite flat surface that has no boundaries.

Platonic solid: The Platonic (regular) solids are convex polyhedra with all faces are one kind of regular polygon and all vertices are identical. They are named after the Greek philosopher and mathematician Plato, who lived around 500 BC. Plato proved that there only exists 5 such shapes; the Tetrahedron, Hexahedron, Octahedron, Dodecahedron, and Icosahedron.

Polygon: A closed chain of line segments. The lines intersect only at their endpoints and no two line segments lie along the same line .

Polyhedron: Three-dimensional geometric shape where each face is a polygon. The plural form is polyhedra. Another name for polyhedron is solid.

Polytope: A geometric figure of any dimension higher than 3.

Prime factor: Any number which is only dividable by itself and by 1.

Prism: A polyhedron whose top and bottom faces are identical and parallel and whose side faces are all parallelograms.

Projection: To cast a shadow so that every point in a given shape has a corresponding point in the shadow.

Proportion: An equality between two ratios (for example: $2/3 = 4/6$)

Pyramid: A polyhedron formed by joining the endpoints of a polygon to a point not in the same plane. The remaining faces formed are all triangles.

Quadrilateral: Any polygon with four sides. All kites, parallelograms, rectangles, rhombuses, squares, trapezoids are quadrilaterals, as are any other convex or non-convex polygon with 4 sides.

Quasicrystal: A type of crystal having a 3-dimensional non-periodic structure.

Ratio: A number obtained by dividing one number into another. This relation is called the ratio between those two numbers (for example: the ratio between 2 and 3 is $2/3 = .666666\dots$)

Rectangle: A quadrilateral whose opposite sides are equal length and whose vertex angles are all 90 degrees.

Red struts: Fit into the pentagonal holes in the node. Structures built in the plane perpendicular to this strut will have 5-fold symmetries.

Regular polygon: A polygon whose edge lengths and vertex angles are all equal.

Regular polyhedron: A polyhedron whose faces are all one kind of regular polygon and all the vertices are identical

Richert / Penrose tilings: Tilings consisting of 2 types of tile, a 72 degree rhombus and a 36 degree rhombus. These tilings can be non-periodic. These tilings were independently discovered by the structural artist Clark Richert in 1971 and by the famous mathematical physicist Roger Penrose in 1975.

Right angle: An angle of 90 degrees.

Rhombi: The plural of rhombus

Rhombus: A quadrilateral whose sides are all equal in length. While diamond shapes are most commonly referred to as being rhombi, the definition also includes squares.

Rotational symmetry: A repetition of a pattern by rotating it around a fixed point by any whole number division of 360 degrees.

Scale: Relative size

Semi-regular solid: See Archimedian solid.

Shadow: Image on a surface created by holding an object between that surface and a light source.

Similar: Having the same shape (same angles etc.) but a different size.

Small rhombicosadodecahedron: One of the 13 Archimedian (semi-regular) solids. It has 12 pentagon faces, 20 triangular faces and 30 square faces. Each vertex has 1 pentagon, 1 triangle and 2 squares. Can be built with blue struts.

Small rhombicuboctahedron: One of the 13 Archimedean (semi-regular) solids. It has 6 square faces, 8 triangular faces and 12 more square faces. Each vertex has 3 squares and 1 triangle. Requires the supplementary green Zome System struts.

Small stellated dodecahedron: A star polyhedron produced by placing 12 pentagonal pyramids on the 12 pentagonal faces of the regular dodecahedron.

Snub cube: One of the 13 Archimedean (semi-regular) solids. It has 12 pentagon faces and 80 triangle faces. Each vertex has 1 pentagon and 4 triangles. Can not be built with Zome System.

Snub dodecahedron: One of the 13 Archimedean (semi-regular) solids. It has 12 pentagon faces and 80 triangle faces. Each vertex has 1 pentagon and 4 triangles. This polyhedron cannot be built using Zome System.

Solid: Alternative name for a polyhedron. Certain families of solids are named for the mathematician or philosopher who is best known for documenting them, such as Platonic solids and Archimedean Solids.

Space frame: In general, a space frame is a triangulated framework made from struts in rigid materials such as steel or wood. Because of the triangulation such structures are extremely strong and light.

Square: A quadrilateral whose edges are equal and whose vertex angles are all 90 degrees. The correct name for the square following geometry naming convention is “regular tetragon”.

Star polyhedron: A non convex polyhedron produced by erecting pyramids on all the faces of a convex polyhedron.

Stellation, Stellated: A process of extending the plane faces and/or line edges of a polygon or a polyhedron until they meet to produce a starry shape.

Surface area: Total area of all the polygon faces of the polyhedron.

Symmetry, Symmetrical: Repetition of a motif by a rotation around a fixed point, a reflection across a line or plane, or by a translation along fixed distances.

Tau, τ : One of two Greek symbols for the Golden section (Divine proportion). This one is used mostly by mathematicians. The exact definition of τ is:

$$\tau = \frac{1 \pm \sqrt{5}}{2} = 1.6180339\dots$$

Tessellation: Comes from the Latin word `tessera` which means tile. A tessellation is simply a tiling of polygons.

Tetra: A Greek prefix meaning 4.

Tetrahedron: A polyhedron with four triangular faces. A total of 64 different tetrahedra can be built with Zome System. The regular tetrahedron, where all faces are equilateral triangles, can only be built with the supplementary green Zome System struts.

3-Dimensional: Any object having length, width and depth.

3-fold symmetry: Repetition of a motif by a rotation of $1/3$ of 360 degrees (120 degrees) about a fixed center point. Examples include: honeycomb, snow flake, clover, the cross section of a green pepper, cucumber, banana, etc.

Translational symmetry: Repetition of a motif by moving it a fixed distance in a fixed direction, infinitely many times if one wishes.

Trapezoid: A quadrilateral that has one pair of opposite sides parallel and the other pair are not parallel.

Tria: A Greek prefix meaning 3.

Triaconta: A Greek prefix meaning 30.

Triacontahedron: A polyhedra shape consisting of 30 rhombic faces. The triacontahedron can be built in Zome System using the red struts.

Triangle: Polygon with three sides and three angles.

Triangular reinforcement: See "triangulation"

Triangulation: Process of making a structure stronger by adding braces that form triangles.

Truncate: To cut off the vertices of a polyhedron.

Truncated cube: One of the 13 Archimedean (semi-regular) solids. It has 6 octagon faces and 8 triangle faces. Each vertex has 2 octagons and 1 triangle. Requires the supplementary green Zome System struts.

Truncated dodecahedron: One of the 13 Archimedean (semi-regular) solids. It has 12 decagon faces and 20 triangle faces. It has 12 decagon faces and 20 triangle faces. Each vertex has 2 decagons and 1 triangle.

Truncated icosahedron: One of the 13 Archimedean solids. It has 12 pentagon faces and 20 hexagon faces. Each vertex has 1 pentagon and 2 hexagons. This is the shape of the Carbon 60 Buckyball molecule.

Truncated octahedron: One of the 13 Archimedean solids. It has 6 square faces and 8 hexagon faces. Each vertex has 1 square and 2 hexagons. This shape can pack with copies of itself to fill 3-Dimensional space without gaps. Requires the supplementary green Zome System struts.

Truncated tetrahedron: One of the 13 Archimedean (semi-regular) solids. It has 4 hexagon faces and 4 triangular faces. Each vertex has 2 hexagons and 1 triangle. Requires the supplementary green Zome System struts.

Truss: A special case of a space frame.

2-Dimensional: Any figure having width and depth but no height.

Vertex: The “point” or “corner” of any polygon or polyhedra. In the Zome System system the vertices are represented by the white nodes.

Volume: A measure, in cubical units of how much 3 dimensional space is occupied by a given 3 dimensional object.

Yellow struts: Fit into the triangular holes in the node. Structures built in the plane perpendicular to the yellow struts have 3-fold symmetries.

Zonogon: A special type of polygon having even number of sides. The opposite sides are parallel and equal in length. An example of a zonogon is a parallelogram.

Zonohedron: A special type of polyhedron having all zonogon faces. The opposite faces are all parallel planes. An example of a zonohedron is a rhombic triacontahedron which has 30 rhombic faces. This shape can be built with Zome System red struts.

Which size kit do I need?

Order kit sizes based upon how many students are likely to use the system simultaneously.

Creator Kit (Teacher Kit – 968 pieces)

The Creator Kit is large enough to do targeted activities with a class of 25 students working in groups. If your students are doing more free building work you will need additional kits.

Includes: 968 precision Zome pieces, 45-page teacher's guide by David Booth, Ph.D., and full-color instruction booklet. The pieces are packaged in re-closable plastic bags in a plastic carry case.

Explorer Kit (Student Kit – 484 pieces)

The Explorer Kit is a good size to use with a pull-out group of 3 to 10 students. It may be the right size if you, for instance, are in charge of a Gifted/Talented program, or do supplementary activities with remedial students. It can also be used as a center activity on a math rotation.

Includes: 484 precision Zome pieces, 45-page teacher's guide by David Booth, Ph.D., and full-color instruction booklet. The pieces are packaged in re-closable plastic bags in a plastic carry case.

Adventurer Kit (Zome 5 – 242 pieces)

The Adventurer Kit is a good size for home use. One to three people can play with it and build medium size structures. For instance, it is possible to build all five platonic solids at the same time with this kit.

Includes: 242 precision Zome pieces and full-color instruction booklet. The pieces are packaged in a sorting tray in a cardboard box.

Pioneer Kit (Zome 3 – 121 pieces)

The Pioneer is a small kit for one person to start exploring with. It is appropriate as a present for one child when you are not sure if they enjoy this kind of activity.

Includes: 121 precision Zome pieces and full-color instruction booklet. The pieces are packaged in a sorting tray in a cardboard box.

Researcher Kit (1,415 pieces)

The Researcher Kit is designed for researchers and other professionals who use Zome System in their work. It has more elaborate packaging and contains a higher proportion of yellow and red struts than the other kits.

Includes: 1,415 precision Zome pieces, 40-page Zome System theory document by Steve Baer, 45-page teacher's guide by David Booth, Ph.D., and full-color instruction booklet. The pieces are packaged in sorter trays separated by Plexiglas dividers in a heavy-duty carry case.